



Error dynamics: the dynamic emergence of error avoidance and error vicariants

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Abstract. Certain kinds of interactive systems can learn to avoid error and develop vicariants for impending or potential error. This paper presents a model of the nature, emergence and development of such error dynamics.

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Some organisms can, in part, avoid error in their interactions with the world, and some can *learn* to avoid error. We have some notions about how this might work, and how we might design artificial systems to do likewise. Such standard notions are, however, arguably limited and bad notions, being based on untenable models of the nature of representation—models of representations as encodings (e.g., Fodor 1987, 1990a, 1990b, 1998)—and, therefore, untenable models of learning (Bickhard 1993, Bickhard and Terveen 1995).

The dynamics of learning about error and of handling error knowledge constitute a complex major theme in evolution and development. Such dynamics range from the simplest forms of learning to the cultural evolution of principles of rationality, as in science. This paper sketches how certain relatively simple kinds of interactive systems can learn to avoid error and can develop vicariants for impending or potential error. I present an outline of the nature, emergence and development of the simplest forms of error dynamics, and describe two more-complex forms as pointers down the evolution of complexity. These constitute progressively more sophisticated versions of the dynamics of evolutionary epistemology (Popper 1959, 1965, 1972, Campbell 1974, Radnitzky and Bartley 1987, Hahlweg and Hooker 1989, Wuketits 1990).

1. Errors and error vicariants

1.1. Avoiding error

The central theme is a progressive elaboration of kinds of dynamics that manage to avoid, detect and, ultimately, to represent error. This paper aims to show how

certain kinds of error detection and error avoidance can emerge naturally in particular kinds of dynamic processes.

Dynamic flow, such as in interaction between a system and its environment, can involve a kind of *anticipation* in the sense that the system can be prepared for some future dynamics, but not for others. Such preparatory anticipation can be false, and falsified, if the actual dynamics encountered violates those anticipations. It is such dynamic anticipations that constitute the most primitive emergence of representation (Bickhard 1993, forthcoming, in press-a, in press-b, Bickhard and Terveen 1995). The central dynamical principle for *this* discussion, however, is that such dynamical error can destabilize dynamics, or destabilize whatever engages in those dynamics, and thereby tend to make it less likely that the dynamical system will enter into that same dynamic realm again. That is, in a system in which dynamic anticipatory error destabilizes the basis for that dynamics, there will be a primitive kind of learning to avoid (the dynamics that yield) those errors. Such destabilization, then, constitutes a kind of simultaneous monitoring for error and yields a learned avoidance of error. The operation of this principle at various levels and in various forms of internal dynamics provides much of the progression of increasingly sophisticated error dynamics. The next paragraphs offer a preview of the next steps in that progression; this is followed by a more careful discussion of those steps.

One crucial step in this progression is the differentiation of interactive from constructive error. This is important because it is useful to substitute some kinds of interactive errors for other errors of interaction that would be more costly, and those substitute, or vicariant, *interactive* ‘errors’ should not count against the *construction* of their underlying interactive organizations—the construction of processes for such less costly error vicariants should not count as constructive error. There are some kinds of error, then—those that are constructed as surrogate or vicariant error detectors in order to be able to avoid more serious error—that should *not* destabilize the relevant processes, and should *not* be (learned to be) avoided by virtue of encountering such error.

Making this distinction requires the introduction of a new dynamics, called microgenesis. This is a special internal dynamics dedicating to setting up the conditions for differing *kinds* of dynamics of interaction between the system and its environment. The basic notion is that distinct functional states or modes in a system do not necessarily correspond to distinct physical regions of the system. Multiple functional conditions, or kinds of conditions, are possible in a single physical region of a system. Thus, one single physical region of a system—such as (a part of) the brain—may function differently from one time to the next. So there must be some process that sets up the new functional conditions, that changes the underlying functional organization and readiness—such a process of preparation for dynamic processes is called *microgenesis* (Bickhard and Campbell 1996).

Recognition of microgenesis as its own realm of dynamics introduces several powerful new dynamic possibilities. Among the most important are the possibilities for *constructive* error vicariants, not just interactive error vicariants. As for the case of interaction, it can be useful to develop vicariants for constructive error that can permit the avoidance of the full cost of actually making the base constructive error. The discussion below will include the development of both *implicit* and *explicit* versions of such constructive error vicariants.

A further level of sophistication occurs when the norms for what counts as a problem and the means for solving that problem are learned simultaneously. That is,

the learning process is self-directed in the sense that the goal, or norm, for the learning is developed as part of the learning itself, instead of being an externally fixed parameter. This sort of learning is essential when the very nature of the problem is itself unclear and must be learned.

Finally, there is the explicit *representation* of error at higher knowing levels. This sets the stage for a full elaboration of (hierarchies of) error knowledge.

1.2. *The emergence of interactive and constructive error*

I call open, far from equilibrium systems that contribute to the conditions for their own existence, such as a flame, *self-maintenant*. A candle flame, for example, maintains above combustion threshold temperature, volatilizes wax and, in standard conditions, induces convection, which brings in oxygen and gets rid of combustion products. Some more complex systems have more than one way to be self-maintenant, and can appropriately select among those possibilities. They can shift their internal processes in ways that shift the kinds of, and manners in which, they are self-maintenant in appropriate response to various environmental changes. Such systems tend to maintain their own condition of being self-maintenant, and I call them *recursively* self-maintenant (Bickhard 1993, 1998, forthcoming, in press-a, Bickhard and Campbell, D. T., forthcoming). A bacterium, for example, might be able to swim so long as it was swimming up a sugar gradient, but tumble if it finds itself swimming down a sugar gradient.

Recursively self-maintenant systems have not only a dynamics of their interactions with their environments—interactions that tend to contribute to their continued existence—they also have an internal meta-dynamics that regulates those basic interactive flows of process, that regulates the shifts among various interactive dynamics. In the bacterium, the dynamics of swimming or tumbling are regulated by the dynamics of switching between them. Such internal regulations constitute the emergence of control relationships and control structures (Bickhard, 1993).

The internal regulatory dynamics of a recursively self-maintenant system—processes that control and modulate the interactive dynamics—will manifest their own dynamic space. The regulatory and the interactive spaces will be coupled, with the *regulatory* dynamics selecting among various alternatives and parameters of the total *interactive* dynamic space, e.g. selecting ‘this’ interactive subroutine rather than ‘that’ one.¹

There is no a priori guarantee that all regions of this dynamic space will be well defined at the level of the regulatory control structures *per se*. They might, for example, move the system through a bifurcation or into a chaotic regime. (They *will* tend to be well defined physically, with the caveat of quantum indeterminism and its potential macro-level manifestations.) In particular, if the interactions move the regulatory process into dynamic regions that are ill defined, they will tend to destabilize the overall dynamics of the system. The critical property here is that instability yields variation: instability in regulatory dynamics will generally yield different regulatory processes the next time that identical or similar conditions obtain. The dynamic spaces are stable at all only insofar as they maintain the stability of the overall system, and ill defined regulations of interactions will constitute ill defined self maintenance processes.

In the extreme, such instability threatens the existence of the system. But more limited versions of such instability can actually *contribute* to that stability, by contributing variation, which, in turn, can contribute to stability. In particular, if

regulatory ill-definedness yields dynamic instability, followed by dynamic restabilization in some new regulatory structure, then cycles of destabilization followed by restabilization can constitute variation and selection constructive processes that progressively construct more regulations. In this simple model, destabilization of dynamics is both a selection against whatever dynamics yielded the destabilization and it is a (presumably blind) variation of that dynamics. That new variation, the new construction, in turn, is subject to further selection if it too should encounter destabilization. In such a dialectic between de- and re- stabilizations, only those regulatory dynamics that succeed in anticipating the interactions and their regulations (in the sense that the dynamics stay within well-defined trajectories in the dynamic spaces) will remain stable.

Destabilization of the dynamics constitutes error. It is a natural error insofar as such destabilization involves risk of system dissolution (although there can be many degrees and kinds of error that are derivative from this primitive version—not all will be so directly associated with system dissolution). In the simple model discussed so far, destabilization is simultaneously error of the *interactive* and *regulatory* dynamics, and it is error of the *constructive* dynamics that generated those interactive and regulatory dynamics. That is, in the primitive case, interactive error and constructive error are ontologically identical.

1.3. *The differentiation of interactive error from constructive error*

It will be beneficial to a recursively self-maintenant system—it will increase its adaptiveness—to develop or evolve dynamics of interaction with the environment whose primary function is to contribute toward the regulation of other interactions. In particular, insofar as the informational redundancies of the environment permit, it will be beneficial to develop interaction forms that can serve as surrogates or vicariants for dynamic errors (Campbell 1974). Vision is a modality of interaction, for example, that is largely dedicated to serving such error vicariant functions. It is much better to encounter the visual error of approaching a wall on your way to the next room than it is to actually bump into that wall: the visual interaction is a surrogate for the collision. The comparison of a visual encounter with a cliff and a physical encounter with a cliff is even more dramatic.

Such a visual encounter constitutes an error of interaction in the sense that it provides information for further guiding and regulating interactions so that the physical encounters can be avoided. Such encounters, such interactions, permit control of overall interaction dynamics in error guided and error corrected ways. But such interaction errors are not necessarily construction errors. In general, in fact, such an interaction error is constitutive of the *appropriate functioning* of the detecting interactive system, and, therefore, the appropriate construction of that interactive and regulatory dynamics. Such interaction errors will not, in general, yield destabilizations of dynamics and consequent restabilizations in new interactive dynamics. That is because such interaction ‘errors’ will remain well stabilized both interactively and regulatively—what to do with and about such visual encounters is (usually) well defined. That is, such interactive error vicariants constitute, among other things, a differentiation between interaction error and constructive error.

1.4. *Constructive error vicariants*

The evolution and development of interactive error vicariants differentiates interactive error from constructive error, but leaves constructive error as the potentially

dangerous destabilization of overall dynamics. It would be useful if vicariants could also develop for constructive error, not just for interactive error—and for the same reason: it's less risky for the system if it can avoid environmental selection effects prior to actually encountering them. How could constructive error vicariants work?

1.4.1. *Microgenetic dynamics.* Before turning to this issue, however, I need to further extend the analysis of interactional dynamics; I need to develop the model of microgenesis introduced earlier. Microgenesis is a form of construction—a *micro*-construction. Constructivism is usually thought of in connection with learning and development: new learning or development is *constructed*, rather than, for example, being passively impressed by the environment into the mind, such as if the mind were a waxed slate. In a standard computer metaphor, constructivism would correspond to the construction of the computer programs that controlled the interactions of the computer with its environment. If a computer could generate new programs for itself, that would be an even closer analogue to constructivism.

In such a computer framework, micro-construction, or microgenesis, is also an applicable notion, though less familiar. In the execution of a program, various registers in the central processing unit of the computer will be shifting among their multiple functional possible modes of functioning. A particular register may be engaged in an integer addition at one moment, and a Boolean exclusive 'or' the next moment. These shifts, of course, will be induced or controlled by the program. The important focus here, however, is on the processes involved in those shifts, in the *set-up* involved in getting a register ready to do an addition rather than an exclusive 'or'. Such a set-up will involve changes in the micro-circuitry associated with the register: a kind of micro-reprogramming of that micro-circuitry to do one thing rather than something else that it is also capable of doing. This micro-programming is a kind of construction—a construction of an 'adder' followed by the construction of an 'or' executor, and so on. That is, it is a kind of micro-construction, or microgenesis.

Genesis, then, is here intended in a constructivist sense, a constructive origin, rather than, for example, origin in a sense of emergence or supervenience. Microgenesis is the micro-constructive origin of the particular micro-modes of functioning or processing in a system as it engages in its overall (interactive) processing.

The important application of the notion of microgenesis for current purposes is to the central nervous system, particularly complex central nervous systems. In *simple* nervous systems, the interactive and regulatory dynamics may be relatively fixed and resident in its entirety in the overall nervous system. Details of those dynamics might be alterable via learning (destabilization and restabilization), but, aside from learning, there are no changes in the interactive dynamic space during interactions *per se*. In more complex nervous systems, however, there can be multiple modes of functioning, multiple dynamics, that a given region of the nervous system, or the entire system, is capable of. In such cases, part of what the regulatory processes guide is the shifting from one mode of dynamic functioning to 'next' modes of dynamic functioning. That is, in more complex cases, the nervous system is capable of multiple (micro-)dynamics, and these multiple dynamics are patched together into the overall dynamic space in a way that is regulated by the regulatory processes (which may themselves be subject to regulation, and so on). This pluripotentiality is at base nothing more mysterious or unusual than the fact outlined above that a

single set of registers in a computer may at one point in time be set to execute an addition, while at a later point the same registers may be set to execute Boolean conjunction. A fundamental difference, however, is that *all* parts of the central nervous system are engaged in microgenesis—*every* part of the central nervous system is akin to a register in a (non-central) processing unit. The frontal lobes of a human being, for example, might at one point be focused on a problem in mathematics, while at another time the same neural system might be focused on an interpersonal issue—and doing different kinds of things accordingly.

What I wish to focus on in such pluripotentiality of dynamic functioning is that the shifting from one mode of functioning to another, the shifting from one region of the overall space of system dynamics to another region of that space, is itself a dynamic process, and will have its own dynamic space. This will also be a kind of meta-dynamics, but a different kind from regulatory dynamics (though they will have to ‘co-operate’ intimately). Such processes of ‘setting up’ the current functional dynamics of the system is the nervous system version of *microgenesis*—microgenesis accounts for the origins, the genesis, ‘on the fly’ of the local dynamic spaces that the nervous system manifests (Hanlon 1991, Bickhard and Campbell, R. L. 1996). Microgenesis, then, is a kind of dynamic process that sets up the overall dynamic system to be able to engage in differing kinds of regulatory and interactive dynamics; microgenesis (ongoingly) constructs the micro-conditions involved in differing kinds of interactive dynamics.

In the computer model version, microgenetic processes construct the momentary dynamics of the central processing unit, and then leave those dynamics to execute while the microgenetic processes either wait for the next need for microgenesis, or move ahead to the anticipated next needed microgenesis. A more sophisticated view of microgenesis in the nervous system would not be restricted to this mutual independence of the microgenetic and the interactive processes, but would at least recognize that nervous system interactive and regulatory dynamic processes, on the one hand, and microgenetic dynamic processes, on the other hand, will both be simultaneously and continuously active, with the ensuing trajectory of microgenesis and dynamic space construction a result of some sort of relaxation processes between the two.²

I have argued elsewhere that, in order for *heuristics* of construction to occur—distinct from strictly blind variation and selection—the variation and selection constructive processes that produce new interactive and regulatory dynamics must take place within the microgenetic processes (Bickhard and Campbell, R. L. 1996). This is in stark contrast, for example, to the computer version in which the processes of constructing computer programs (programming) are completely distinct from the microgenetic processes of setting up the CPU. The basic intuition of the necessary intimate relationship between microgenesis, on the one hand, and the constructions of learning and development, on the other, is that the stabilized setting up of *old* and successful dynamics (microgenesis) must occur *in the same process* as the variational setting up of *new* trial dynamics (heuristic learning, development, problem solving) in order for the *constructive* ‘location’ of the *successful* constructions to be available to heuristically guide the microgenetic, micro-constructive, processes of *new trial* constructions. Trial solutions to *new* problems must be in some sense ‘near’ to well-established solutions to *old* problems—to which the new problems are ‘similar’. Modeling how all of the information involved in such issues of ‘similarity’ and

'nearness' can be functionally available to actual problem solving heuristic processes is decidedly non-trivial (Bickhard and Campbell, R. L. 1996).

What I will be making use of for further discussion from this notion of microgenesis is:

- that microgenesis is itself a realm of dynamics, with its own dynamic space, and
- that basic destabilizations and restabilizations are processes that occur *within* the processes of microgenesis, thus yielding changes in what is microgenetically constructed.

Microgenetic destabilizations are manifestations of regions of microgenetic dynamics that are indeterminate or chaotic, and stabilization is constituted by the establishment of well-defined organizations of microgenetic dynamics.

1.4.2. *Implicit constructive error vicariants.* Returning now to how constructive error vicariants could function and emerge—and, therefore, how they could exist at all—consider first a kind of *implicit* constructive error vicariant that emerges from a constraint that is inherent in microgenesis. This constraint, in fact, is intrinsic to the nature of variation and selection processes. It is especially so when the processes of variation and selection are themselves subject to variation and selection—as they are in evolution—and when variation and selection of interactive dynamics occurs via variation and selection on the microgenesis of those dynamics.

The basic recognition is that the space of possible microgenetic constructions will change as the dynamics of microgenesis change. One way in which such changes could occur is for particular regions of possible microgenetic dynamics, and, thus, of the constructions that might be set up, to become relatively unreachable by those microgenetic dynamics. If few possible microgenetic trajectories enter or terminate in such regions, then they become less likely to occur at all. If previous microgenetic dynamics that traversed such regions strongly tended to encounter their own errors—destabilizations—then the processes that traversed those regions become less likely in the future. Such regions of microgenetic dynamics, therefore, will tend to either remain regions of destabilization of microgenesis or to become such if they weren't before. Microgenesis, then, will tend to avoid such regions.

The process of learning a physical skill might provide human examples. The progressive approximation toward acceptable physical motions and strategies that is involved in skill learning is dual to a progressive *avoidance* of the large dynamic subspace of motions and strategies that don't work at all. That avoided subspace will become microgenetically unstable, so that learning trials don't even attempt to explore it (after sufficient initial experience). This can be dramatically evident if that failure subspace can yield explicit destabilizations, such as pain, in addition to the failure *per se*. One example might be the avoidance of physical motions that get too close to a hot engine while learning to repair some part of it.

Regions of intrinsic microgenetic instability, by virtue of that instability, will constitute implicit criteria against the error of entering such a region: they will be implicit in the sense that there is no explicit detector for error, and certainly no explicit representation of the properties that count as error. Nevertheless, such regions can constitute, and can be learned as, implicit vicariant guides to the avoidance of constructive, microgenetic, errors.

1.4.3. *Explicit constructive error vicariants.* In a large nervous system, microgenesis will be ongoing simultaneously throughout the system, just as will interactive and regulatory dynamics. Microgenetic dynamic space, then, will not only manifest structure of the possible dynamic trajectories of the process through time, it will also manifest structure of the possibilities of simultaneous microgenetic process in differing regions of the central nervous system. Simultaneous microgenetic processes, in turn, yield the (emergent) possibility of internal interactions among the microgenetic processes themselves.

From an evolutionary perspective, differentiations of microgenetic processes, partial modularizations, will occur out of a simpler framework of more global undifferentiated processes. Differentiated processes of microgenesis, then, will tend to influence each other—they will have been differentiated out of a common process—and will *of necessity* influence each other if their microgenesis activities are to remain coordinated throughout the nervous system. Once the possibility of concomitant microgenetic processes emerges, it becomes possible for some processes to *monitor* others.

One mode of influence will be for one process—a monitoring process—to discriminate among various dynamic possibilities in another process. That is, some microgenetic dynamics in a monitored process would influence the monitoring process to proceed in one way in its own dynamics, while other microgenetic dynamics in the monitored process would yield different processes in the monitoring process. The monitoring process would, via such differing modes of being influenced, differentiate one class of dynamics of the monitored process from other classes of possible dynamics.

If one or more of those classes of possible differentiations should evoke from the monitoring process a destabilization of the monitored process, then the monitoring process is more than a monitor: it is a selection constraint operating against the destabilized microgenetic dynamics. The variation and selection processes that create such a monitoring selection constraint will tend to create and maintain such constraints only when they tend to be functional toward system stability—when they tend to guide microgenetic constructions away from potential construction errors.

Microgenetic monitors of microgenetic processes that can destabilize those processes when they are risking error, then, constitute *explicit* vicariants for constructive error. Such a monitor, for example, might catch a motion that would risk touching a hot engine even though that motion was being generated by a microgenesis (of a kind of motion) that hadn't been tried before, so that no portion of the microgenetic space would as yet be microgenetically destabilized. That is, it is possible for explicit error vicariants—monitors—to catch microgenetic errors even in microgenetic spaces that have yet to be explored, and, therefore, spaces that could not have as yet developed implicit error vicariants: explicit error vicariants can be more powerful than implicit error vicariants.

The emergence of implicit and explicit constructive error vicariants are the first steps in a complex evolution and development of increasingly sophisticated error dynamics. The dynamics of microgenesis is what generates these two forms of constructive vicariants, and it plays a central role in the more sophisticated developments as well. The focus of this paper is on these initial steps in error dynamics development, on getting the initial steps outlined, and on highlighting the importance of microgenesis in such processes. I will, however, point to two further

steps in the sophistication of error dynamics, by way of indicating where a dynamic analysis proceeds from here.

2. More complex error dynamics

2.1. *Self-directed anticipatory learning*

The emergence of constructive error vicariants, implicit or explicit, constitutes a beginning differentiation of interactive error from constructive error. Constructive error vicariants help guide the system away from anticipatable constructive errors, and, thus, from regions of likely interactive error.

But constructive error vicariants as discussed to this point are relatively fixed and inflexible. They emerge and monitor. A more powerful form of normative differentiation occurs if error vicariants for construction and interaction progressively differentiate from errors of interaction *per se* in the course of learning itself. That is, if the system can learn more about what the problem is simultaneously with learning more about how to solve the problem.

We know that such sophisticated learning occurs—for example, a detective learning more about what kind of evidence is likely to be relevant, and, therefore, should be sought, as he learns more about what sort of crime and suspect is involved. The problem is to account for it. Christensen has dubbed such learning as *self-directed anticipatory learning* or SDAL (Christensen and Hooker 2000, Christensen and Bickhard, in preparation). It constitutes a next step in sophistication beyond constructive error vicariants *per se*.

2.2. *Epistemic error vicariants: reflection on error*

Another major advance in error dynamics is enabled by the possibilities of reflective consciousness. Reflective consciousness, of course, poses its own fundamental problems of nature, evolution, emergence, development, and so on (Campbell and Bickhard 1986, Bickhard, in press-b). The crucial characteristic for current purposes, however, is that it permits the ability to represent errors and kinds of errors explicitly, and, therefore, to think and reason about them. Elsewhere, I argue that reflective hierarchies of knowledge of kinds of error, with higher levels being about lower levels, constitutes the skeleton of rationality (Brown 1988, Bickhard 1991, in preparation, Hooker 1995). Rationality, in this view, is generated as the most sophisticated error dynamics available, making full use not only of microgenetic error vicariants and self-directing differentiations of norms and procedures, but also the power of reflection and the sedimentation and historicity permitted by social language.

3. Conclusion

Error dynamics range in complexity from simple behavioural responses to error, to vast ranges of sophistication in learning to discover error and to avoid error, to hierarchies of reflection and reasoning about possible error. I have shown how microgenesis plays a crucial dynamical role in the simplest kinds of learning to avoid error. In particular, I have shown how properties of microgenetic destabilization and restabilization, and dynamical interactions among concurrent microgenetic processes, can generate implicit and explicit error vicariants, thus permitting an organism more sophisticated error avoidance—more sophisticated and adaptive

error dynamics. Self-directed adaptive learning is a still more sophisticated process of developing knowledge about what counts as solving, or failing to solve, a problem, simultaneously with the processes of learning how to solve it. And reflective consciousness permits hierarchies of error knowledge about error knowledge, and of reflective and social dynamics with respect to such knowledge. Such explicitly represented error knowledge and error dynamics constitutes the skeleton of rationality.

Notes

1. The regulatory dynamics could couple with the leaves of a foliation of the total interactive dynamic space, e.g. a coupling with the parameters of the foliation, if the mathematical conditions of a manifold are satisfied (Marmo *et al.* 1985, Tamura 1992, Kolar *et al.* 1993, Candel and Conlon 2000). A discrete dynamics, on the other hand, such as a typical programming language provides, would not in general manifest a dynamical *manifold*, and, therefore, not a coupling via a foliation. Nevertheless, a discrete dynamics could manifest a meta-dynamics that would regulate (control) a system of directly interactive routines. That is, the distinction between an interactive dynamics and a regulatory dynamics could still be made, for example, in terms of a distinction between interactive processing and control flow.
2. A still more sophisticated view would recognize that such a 'relaxation' process is more likely to be a process of mutual selection constraints between interactive dynamics and microgenesis. Mutual constraining relationships among endogenously active processes permit greater flexibility than 'simple' relaxation processes, and altering parameters of those mutual constraint processes is still another source of flexibility: mutual selection constraints is a more complete use of the power of variation and selection than is relaxation (Bickhard and Campbell, D. T., forthcoming).

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