2 The Import of Fodor's Anti-Constructivist Argument

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Fodor argues that the construction of genuinely novel concepts is impossible and, therefore, that all basic concepts available to human beings are already present as an innate endowment (1975, 1981). This radical innatism - along with related conclusions such as an innate modularity of available representations and a corresponding innate limitation in the potential knowledge that human beings might be capable of (1983) - has been seen by many as a reductio ad absurdum of Fodor's position, and his arguments have consequently been dismissed. I will argue that Fodor's arguments deserve much more careful attention than that: in particular, his arguments *are* a reductio of one of his essential presuppositions, but it happens to be a presupposition that he shares with virtually all of psychology and philosophy. Fodor's conclusions, then, are reductios of the major portion of contemporary studies of cognition and epistemology (Campbell and Bickhard, 1987). Furthermore, even when the critical presupposition is isolated, it is difficult to construct a genuine alternative. Most attempts at correcting any part of the logical difficulties involved have inadvertently presupposed the pernicious premise elsewhere in the system (Bickhard, 1980a, 1982, 1987).

Fodor's Argument

Fodor's arguments are usually stated in terms of theories of learning or concept learning and yield the conclusion that there are in fact no such theories: what claim to be theories of learning are really theories of belief fixation (Fodor, 1975, 1981; Piattelli-Palmarini, 1980). Fodor's basic point is that all such theories require that concepts or hypotheses be already present before learning can begin; 'learning' is then limited to establishing - or failing to establish - belief in a preexisting concept or hypothesis. The new concept, the new representation, must be already constructed before learning begins. (This is just Popper's argument against the logical possibility of passive induction [Popper, 1959, 1965, 1972, 1985], but Fodor seems unaware of that, or unwilling to acknowledge it. He even persists in calling such hypothesis testing "induction.")

According to this argument, then, learning cannot create new representations. The representations that learning is supposedly concerned with arise not from learning itself, but from constructions of structures of prior representations. This is a combinatoric type of model in which new combinations of already present representations may be assembled and then tested against experience via "induction." Such a view, however, requires that there be other representations already present that can participate in the combinations, and although some of those may themselves be products of still earlier combinatory constructions, there must be some basic ground of representations that suffices for all possible such combinations. The alternative is an infinite regress of combinations of ever more primitive representations. Because this would be a regress of actualities - of actual representations, supposedly in the mind - it is unacceptable.

By assumption, the basic representations cannot themselves be decomposed into combinations of lower order representations, but no models of learning or development provide any way for such fundamental representations to be created. Therefore, Fodor reasons, these basic representations must be innate, i.e., provided in the genome. The conclusion is that some basic set of representations, combinatorically adequate to all possible human cognitions, must be innately present.

Fodor presents a number of related arguments that I will not pursue directly, but that depend in part on the innatist conclusion of the above argument, so I would like to indicate the nature of that dependence. First, because there is little evidence for any combinatorial complexity in studies of lexical comprehension, Fodor concludes that any combinations from innate primitives that do exist are relatively shallow. The result is that the resources of the innate base must be very strong indeed, because even the power of unrestricted combinatorics seems to be unavailable to the constructive processes founded upon it (Fodor, 1981). Second, along with other arguments, the assumption of an innate base for all possible human representation yields as a plausible conclusion that human representational (and inferential) capacities are clustered in innate modules (Fodor, 1983). Third, the representational power offered by such an innate base of representational atoms is at the same time a restriction of power: anything that would require a representation that can*not*, in principle, be combinatorically constructed from those atoms is intrinsically beyond human capacities to represent or understand (Fodor, 1981, 1983). I mention these subsidiary arguments not in order to address them in detail, but rather to indicate their relationship to the basic innatist argument and, therefore, to indicate the sense in which they, too, are undermined should Fodor's basic argument prove to be flawed.

Reconstructing the Argument

Stating the argument in terms of learning or development is, I suggest, somewhat misleading. It is misleading because it directs attention to theories of learning or development as being the source of problems (should the argument be taken as revealing a problem, rather than just as explicating the way of the world). The more fundamental source of the impasse which Fodor's argument reaches, however, is not located in theories of learning or development per se, but instead in the theories of representation that are presupposed by those approaches to learning and development - and by virtually all the rest of cognitive and epistemological studies as well.

The core of the difficulty is that the only way that these notions of representation allow for new representations to be constructed is as combinations of elements that are themselves *already representations*. The rules of combination may vary enormously in differing models, as may the presumed base of representational atoms (should the model be so explicit as to acknowledge and address the necessity for such a representational periodic table), but the presupposition of "representations only from representations" holds universally. After all, "out of nothing nothing comes" (Fodor, 1975, p. 59).

But this implies that one must already have representations in order to construct representations, and that implies, in turn, that it is impossible for representations to be constructed, to come into being, out of foundations that are not themselves representational. It follows, then, that it is impossible for representations to emerge out of non-representational phenomena or processes. It is this impossibility that is at the heart of Fodor's anticonstructivism: learning and development are incapable of creating *emergent* representations; they can only create new combinations of old representations. The origin of this seeming impossibility, however, is in the notions of representation involved, not directly in the models of learning or development.

In effect, then, Fodor's arguments have turned on the inability of contemporary studies of cognition and epistemology to account for the *emergent nature* of representation. Such studies account at best for the combinatoric nature of *non*-emergent representation, but they are thereby vulnerable to Fodor's challenge.

If representations cannot emerge, however, then they cannot come into being at all. A narrow focus on this point yields Fodor's innatism: neither learning nor development, as currently understood, can construct emergent representation; therefore the basic representational atoms must be already present genetically. Unfortunately, this conclusion does not follow. If representation cannot emerge, then it cannot emerge in evolution any more than it can in development. The problem is logical in nature and is not

specific to the individual. Conversely, if some way were posited in which evolution *could* yield emergent representation, then there is no a priori reason why that emergence would not be just as available in the development of the individual. Fodor's innatism, then, simply misses the basic issue. If representation cannot emerge, then it is impossible for it to exist, and evolution is in no better position in this respect than is individual development; on the other hand, if representation *can* emerge, then there is something wrong with the models of learning and development that cannot account for that emergence. When those models are corrected, that emergence should be as available to the individual as to evolution. In either case, Fodor's strong innatism does not follow. (In this final innatist conclusion, then, Fodor's reasoning is *not* valid.)

Clearly, representations do exist, and, therefore, representation *can* emerge from non-representational phenomena. Therefore, equally clearly, there *is* something wrong with contemporary models of representation that make that emergence logically impossible. Fodor's anticonstructivism is a reductio of models of representation that make the emergence of representation logically impossible, but, ipso facto, Fodor's innatism is, therefore, not a viable solution to the problem.

What's Wrong with Contemporary Models of Representation?

Fodor's anticonstructivism turns on a basic flaw in contemporary models of representation: the inability to account for the emergence of representation. This is an abstract indictment, however, which still leaves open the question of *why* contemporary models are subject to this charge. In this section, I will argue that the root problem is in the assumption or presupposition (it's not always explicit) that representations are constituted as some sort of encodings (Campbell and Bickhard, 1987).

Paradigmatic encodings are such schemes as Morse Code, in which, for example, "..." stands for "S." Morse code and other similar codes, such as computer codes, security ciphers and codes, and so on, are unproblematic with respect to the issues at hand, but the very nature of their "unproblematicness" reveals some of the deep sources of difficulties in more epistemologically ambitious encodingisms. The encoding, "..." say, in such a scheme is explicitly paired with another representation that it is to be taken as standing in for-"S" in this case. The encoding correspondence is explicitly defined, and that which the correspondence is with-"S"-is itself an unproblematic representation of the sound /s/. All actual encodings are constituted by explicit known correspondences with known defining representations. It is only when it is assumed or presupposed that such a model could hold for epistemology in general that fatal difficulties emerge.

Encodings, fundamentally, are stand-in representations. In Morse code, "..." stands in for "S" and picks up the same representational content that "S" carries, namely /s/. "S" provides that content by already carrying it prior to the definition of Morse code, and "..." is established within Morse code precisely by being paired with "S" as a stand-in. Note that "..." obtains its representational content from "S," and "S" is presumed to *already* carry *its* representational content. Nowhere in the nature of an encoding is there any possibility of a *new* representational content being established, except in the purely combinatoric sense of an encoding element being taken to stand in for some combination of previously established elements. Encodingisms, then, involve precisely the inability to address the emergence of representational content upon which Fodor's arguments turn.

For another perspective on this issue, consider the definition of an encoding as being an element with a known representational content. This definition is, in fact, equivalent to the 'stand-in' definition just mentioned. In one direction of implication, a stand-in definition *provides* a known representational content to the defined element, thereby *making* it an element with a known representational content. In the other direction, any specification of representational content for the definition of an encoding element requires some other representation(s) which can provide or specify that content, and the new element then becomes a stand-in for whatever representation(s) provided that content. This definition, however, makes it clear that any representation that is constituted as a representation by virtue of the representational content which it carries is intrinsically an encoding. Any representation defined as a representation in terms of what it represents is an encoding.

But a representation defined as such in terms of what it represents *cannot* address the issue of representational emergence; in such a scheme, representational content must already be available before any encoding representation can be defined. There is no starting point, no origin, for the definitional process. If it is presupposed that encodings *can* somehow create new representational content - can provide the foundations for their own definitions - a logical incoherence results. In particular, any finite number of stand-in definitions is unproblematic: they are "simply" passing on already available representational contents. But when we address the foundational representations upon which others are to be defined, impossibilities emerge. If such foundational representations are presumed to be encodings and if we ask how those foundational encodings are provided with representational content, no answer is possible. Put another way, if we ask how an epistemic agent could possibly know what a foundational encoding is *supposed* to represent, no answer is possible. The point is made simply by noting that, on one hand, if the foundational encoding is provided with content in terms of any other representations, then it is not foundational, contrary to hypothesis.

On the other hand, if such a foundational element, say "X," is presumed to provide its *own* content, we have ' "X" represents whatever "X" represents' or '"X" stands in for "X." ' In neither case is any representational content established, and, therefore, in neither case is "X" established as an encoding at all. Consequently, the notion of a foundational encoding is logically incoherent. Presupposing such a foundation leads to the incoherent conclusion of an encoding element with no representational content.

This incoherence is, in fact, just the *logical* aspect of the inability of encoding models to address the emergence of representational content. Such emergence, if it is going to occur anywhere, will have to occur at the foundations of a strict encoding scheme, and, in fact, it *does* have to occur *somewhere* if any version of encodingism is to "get off the ground." But any actual encoding scheme is only capable of providing and carrying representational content, not creating it. Consequently, the presuppositions of an encodingism necessarily include the incoherent presupposition that foundational encoding elements are possible, that encodings can, by themselves, do something that they cannot do: create new representational content.

Why Should We Care?

One deeply important consequence of this incoherence of encodingisms, of this inability of encodings to account for new representational content, is that encodings *cannot* be the answer in any case where *new* representational content is required. Simply stated, encodings cannot cross epistemic boundaries. Encodings cannot provide basic representations for basic domains of knowledge, they can only change the form of representations in or across *already represented* domains of knowledge. In particular, encodings cannot provide the basic representations for the mind about its environment - whether perceptual or otherwise (Bickhard and Richie, 1983). Similarly, they cannot provide a basic representational access from the world - language, say - *into* a mind (Bickhard, 1980a, 1987). In neither case can encodings create the foundational representational contents required to start the process of representing. *Encodingisms, in other words, are logically incapable of ANY of the basic epistemological tasks for which they are standardly proposed.*

That fundamental inadequacy is the logical fulcrum for Fodor's arguments, and, consequently, we ignore those arguments at our peril. They point to a deep flaw in contemporary epistemology that is not at all easy to avoid - virtually *all* available models of representation are encodingisms (Bickhard, 1980a; Bickhard, 1982; Bickhard and Richie, 1983; Campbell and Bickhard, 1986; Campbell and Bickhard, 1987; Bickhard, 1988; Bickhard and Campbell, 1989; Bickhard and Terveen, 1990). The proper conclusion from Fodor's arguments, however, is not one of a radical nativism. Nativism and

anticonstructivism, in fact, manifest only partial and distorted insights into the fundamental problem. It is, rather, "that the argument has to be wrong, that *a nativism pushed to that point becomes unsupportable, that something important must have been left aside*. What I think it shows is really not so much an a priori argument for nativism as that *there must be some notion of learning that is so incredibly different from the one we have imagined* that we don't even know what it would be like as things now stand" (Fodor in Piattelli-Palmarini, 1980, p. 269). Actually, as mentioned, not quite, the fundamental problem lies in our notions of representation, not in our notions of learning.

In-principle arguments concerning the inadequacy of a modeling approach, such as the argument concerning encodingism, can at best be disquieting unless there is a plausible alternative. Without an alternative, it is difficult to know what to try to correct and what implications such a correction might have. Even at the level of disquiet, however, I wish to emphasize that Fodor's arguments, along with, most importantly, the presuppositions upon which they turn, should be cause for great unease. The basic encodingist assumption of *all* contemporary approaches to cognition is fundamentally incapable of solving *any* of the basic epistemological problems with which we are concerned. The resulting anticonstructivism, nativism, intrinsic limitations to knowledge, and so on, are all very much secondary to that basic difficulty. Furthermore, with so foundational a flaw in our most central presuppositions, it is not at all clear what sorts of guidance from the resultant models we can trust in perception, cognition, language, learning, development, education, instruction, or any other domain involving representation. I do not wish to leave this discussion only at the level of disquiet, however. There is an alternative approach to the nature of representation, called interactivism, and I will give a brief and limited introduction to it here (Bickhard, 1980a; Bickhard and Richie, 1983; Campbell and Bickhard, 1986).

Interactivism: Outline of a Solution

The incoherence of encodingism originates in a basic circularity in the notion that an element can be an encoding only if it carries representational content, yet encodings are taken to be the only source of representational content. When pressed at the level of foundations, then, encodings must provide their own contents in order to be encodings at all, but in order to provide such contents, they would already have to be encodings. There is no way out of the circle and, thus, no way to avoid the incoherence.

Interactivism avoids this circularity and resulting incoherence by separating the condition of being a representation from the condition of carrying representational content. It becomes possible for representational

elements to come into being *without carrying any representational content* and only subsequently for representational content to be given to them. This breaks the circularity and thus avoids the incoherence. This abolishes the requirement that we must already have representations in order to get representations.

It does so, however, only by providing a definition of representation that is more primitive than that of "carrying representational content," but that can, once extant, address the problem of representational content. The problem for interactivism as an alternative, then, is twofold: first, provide a notion of representation that does not require the carrying of representational content; and second, provide a notion of representational content that can be constructed subsequent to the construction of a basic representation and that can be carried by it.

The foundational conception here is that of interactive differentiation. First, an *epistemic* system is argued to intrinsically be an *interactive* system (Bickhard, 1980a, 1988; Bickhard and Richie, 1983; Bickhard and Campbell, 1989). Knowing is successful interacting, and knowledge is the capability for knowing. System processes of knowing, of successful interactions, provide the non-representational foundations for explicating representation. The internal course of process in an epistemic system will depend jointly on the organization of the system and on the environment engaged in the interaction. Consequently, the final state in which that system (or subsystem) is left when the interaction is completed will also depend jointly on the system and on the environment. Each possible final state of such an interactive system or subsystem will correspond to the class of possible environments defined by the condition that each environment in the class would yield that final state in an interaction. Each final state, then, *differentiates* its class of environments from those environments that would yield some different final state. This notion of interactive differentiation is more primitive than the notion of carrying representational content. A final state differentiates its class of possible environments from all others, but *it doesn't represent anything at all about those environments except (implicitly) that they yield this final state rather than some alternative final state*. There is no representational content here, only a functional and implicit differentiation.

Interactive differentiation, then, does provide a notion of representation that is sufficiently primitive as to not require representational content. The next step is to model how representational content could be provided for those environments that are "contentlessly" differentiated. The core intuition for this step is to realize that environmental differentiations may be useful for the system in further differentiating its own internal activities. Environments of the sort that yield final state "A" may be amenable to strategy "S22," while environments of the sort that yield final state "B" may be more appropriate to

strategy "S87." But such knowledge implicit in the functioning of the system *is* representational content. If the system can select the appropriate strategy given an environmental differentiation, that *constitutes* representation of a property or properties of that class of environments. It constitutes representation of the implicit predication that "final-state-A type environments" are also "appropriate-to-strategy-S22 type environments." In this way, a final state can come to acquire representational content, perhaps vast content in complex organizations and webs of such selectional properties. Further, once given representational content, an interactive representation can serve to provide representational content for the definition of a *derivative* encoding; encodings can be defined as stand-ins for interactive representations.

Interactivism's claim to be an alternative to encodingism and one that avoids encodingism's tangle of incoherences and absurd consequences is a programmatic claim. It is programmatic in the sense that interactivism is a general approach to epistemological phenomena whose potential can ultimately be demonstrated only by constructing useful particular models within that general interactive framework. There will be little programmatic elaboration here, so interactivism will be presented as a general programmatic approach based on in-principle arguments. More extensive elaborations of interactivism can be found in Bickhard (1980a), Bickhard (1980b), Bickhard and Richie (1983), Campbell and Bickhard (1986), and Bickhard (1987).

There are two consequences of interactivism, however, that I will briefly present because they are responses to prima facie serious challenges to interactivism's general programmatic claims. The first challenge is simply to turn the root of the encodingism incoherence problem back to interactivism: How can interactivism account for the emergence of novel representation, of representation constructed out of non-representational foundations? The general form of the response is that interactivism logically forces a *constructivism*, and a constructivism that does in fact provide a solution to the problem of representational emergence. The second challenge is to the general representational adequacy of interactivism. Interactive representation emerges as a functional aspect of goal-directed interactive systems. Even if the direct claims of interactivism are taken at face value, it might still seem that interactivism could at best account for representation of physical environments. Interactive representation requires interaction, and interaction requires something to interact with, and (so the challenge goes) it is not clear what the interactive environments could be for abstract knowledge such as that of mathematics. The general form of the response to this challenge is to show that interactivism yields a hierarchy of levels of potential knowledge in which each level is of increasing abstraction relative to the levels below.

I turn first to the interactive account of emergent representation. The temptation within encoding models is to try to account for the creation of representations via the passive creation of element-by-element and structural correspondences with things and patterns in the world. Encounters with the world will result in things, events, and patterns in the world impressing themselves into a receptive mind, thereby creating encoding correspondences, the presumed impressions, with those things, events, and patterns. The classic version of this is the tabula rasa, the blank waxed slate, but contemporary notions of transduction and induction are simply more sophisticated versions of the same idea (Bickhard and Richie, 1983). Unfortunately, even if the basic passivity of mind in these models is accepted and even if the passive creation of such correspondences is also accepted, an encoding is constituted as a *known* correspondence, known both in terms of the fact of such a correspondence and in terms of what the correspondence is with. The incoherence argument shows that the origin of that knowledge *about* the correspondences is precisely the fatal inadequacy of encodingism. Factual correspondences are easy to find, they occur in physics all the time; it is knowledge that is difficult to account for.

Interactivism, in contrast, does not offer the same temptation. Interactive representation is not a matter of correspondence relationships at all but, rather, an emergent of interactive functional relationships. There are *no* particular element-by-element or structural relationships between an interactive differentiating system and the environments which it serves to differentiate. There are only open, functional differentiating relationships between a system and its environment. The system differentiates, but it does not correspond.

The final states might be said to correspond to their differentiated classes of potential environments, but this can only be specified from the perspective of an external observer of the system who independently *already has* representations of *both* the system's final states *and* of the environments that are differentiated, and who can therefore *define* the correspondences between them. Such correspondences can be defined within the *observer's* perspective on the open system differentiations, but the *system itself* has no representations of such correspondences, nor even necessarily of their existence, and they can play no direct epistemic role for the system (cf. Maturana and Varela, 1980, 1987; see Bickhard and Terveen, 1990). The fact, however, that system differentiations can be described by an external observer in terms of correspondences is one more reason why encodingism has enjoyed such credence in spite of its logical difficulties.

Interactive representation is a matter of a system's functional relationship with its environment, not a matter of structural relationship. Any notion of the environment creating a structural relationship, therefore, is irrelevant. Furthermore, there is no possibility that the environment can

passively impress or create a *system*. Interactive representation, then, can be created only by the *internal constructions* of the system itself. The system must try out new system organizations and test them against the environment, a variation and selection constructivism.

Still further, those internal system constructions are of system organization, *not of representations*. The construction of interactive representations is emergent in the construction of system organization and *does not occur* (except with respect to secondary encodings) in terms of combinations of already available representations. A solution to the problem of representational emergence, then, is intrinsic in the nature of interactive constructivism. System constructions construct new system organization, and representation emerges as a functional aspect of system functioning, so already existing representations are not required.

I turn now to the second challenge, concerning the interactive account of abstract knowledge. The basic intuition for the problem of abstract knowledge is fundamentally Piagetian: the properties and activities of an interactive system are more abstract than the environmental properties being represented by that system. We begin, then, with a first-level system that interacts with and represents its environment. This system, in turn, will have properties that could be represented by a second-level system interacting with the first level system. The first-level system would be the interactive environment for the second-level system. The second-level system, in turn, has still more abstract properties that could be represented from a third level, and so on indefinitely. Interactivism, in this manner, generates an unbounded hierarchy of potential levels of representation, of potential interactive knowing, of ever-increasing abstraction. Interactivism, then, does not face an impasse with respect to abstract knowledge, but instead has unboundedly rich resources for addressing abstractions.

Furthermore, these levels of potential knowledge are not merely an ad hoc solution to the problem of abstraction, but instead have many other properties that connect with other phenomena in psychology and epistemology. For example, constructivism forces the levels of knowing to be ascended one at a time, in sequence, from the first level upward. It is impossible for any system to exist, to be constructed, at a given level unless there is already something to be interacted with at the level below. The hierarchy of levels of potential knowing, then, forces an invariant sequence of stages of constructive development. These stages, in turn, account for the major qualitative shifts found in development (Campbell and Bickhard, 1986). The levels, in other words, do far more than just account for abstract knowledge.

For another example, I argue that the *constructive* relationship between one level and the next higher level is essentially that of Piaget's notion of reflective abstraction (Campbell and Bickhard, 1986), while the *epistemic*

relationship between one level and the level below is that of reflective consciousness (Bickhard, 1980a, 1980b; Campbell and Bickhard, 1986). Interactivism, then, by forcing constructivism and levels of potential knowing, accounts in a natural way for the related phenomena of reflective abstraction and reflective consciousness. In a similar manner, many other phenomena of psychology and epistemology emerge via natural elaborations of the basic interactive approach.

Interactivism, then, does not fall prey to the incoherences of encodingism, but rather offers a rich alternative perspective on the phenomena that are traditionally conceded to encodingism. In the context of this discussion, the critical point is that interactivism does offer a way out of the impossibilities and absurdities presupposed by and revealed in Fodor's arguments.

Conclusions

Fodor's arguments appear to reach absurd and unacceptable conclusions. On that basis, it can seem all too easy to dismiss them. I have argued that Fodor's conclusions *are* absurd and unacceptable, but that his arguments are founded on a fundamental flaw in contemporary approaches to the nature of representation. If that is the case, then it is logically inconsistent, though not trivially so, to both reject Fodor's arguments and accept contemporary approaches to representation. The arguments follow directly from the encodingist approaches, and encodingism is ubiquitous in those contemporary approaches. Still further, encodingism is extremely difficult to abandon, both because there is no readily available alternative and because many positions have deep, non-obvious logical commitments to an encodingism. It is not easy to uncover and replace implicit encodingisms (Bickhard and Richie, 1983). I have briefly outlined an alternative to encodingism called interactivism. Interactivism is not susceptible to the impossibilities and absurdities of encodingism. The replacement of encodingism and its equivalences by interactive models will be a slow, programmatic process, but encodingism itself is epistemologically impotent and incoherent and, therefore, *must* be replaced.

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