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Piaget on Variation and Selection Models: Structuralism, Logical Necessity, and Interactivism[1]

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Abstract. Throughout his career, Piaget rejected the adequacy of random trial and error or variation and selection models. Instead, he argued that teleonomic autoregulations were necessary to account for the facts of evolution and development. This purported necessity of teleonomy has been a controversial and generally rejected aspect of his model - especially in its evolutionary version. Necessary teleonomy, however, is not an isolated part of Piaget's thinking, but is instead deeply motivated by two central forces throughout Piaget's oeuvre: a complex of assumptions organized around his structuralist assumption concerning the nature of knowledge, and the centrally important epistemological problem of logical necessity. Structuralism, however, is shown to be a seriously flawed foundation for Piaget's epistemology, and to be at the center of a number of inadequate and erroneous positions within Piaget's writings - positions concerning epistemology, evolution, and even necessity itself. An alternative conception of knowledge is outlined - interactivism - that offers a corresponding alternative approach to necessity.

Throughout his career, Piaget rejected the adequacy of random trial and error, or variation and selection models [Piaget, 1952, 1966, 1971b, 1980, 1985; Piattelli-Palmarini, 1980]. Instead, he argued that teleonomic regulations were necessary to account for the facts of evolution and development. Because this purported necessity of teleonomy has been a controversial and generally rejected aspect of his model - especially in its evolutionary version - it is of interest to understand more thoroughly why Piaget

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held to it, and what relationship it has to other aspects of his thinking.

We shall find that this is not an isolated and aberrant part of Piaget's thinking, but is instead deeply motivated by two central forces throughout Piaget's oeuvre: a complex of assumptions organized around his structuralist assumption concerning the nature of knowledge, and the epistemological problem of logical necessity. In simple preview, unguided variation and selection constructivism is not sufficient to account for evolutionary or developmental emergence of necessary knowledge as long as knowledge is construed in structuralist terms.

Piaget is one of the few psychologists to have taken seriously the epistemological problem of necessity. Logical necessity imposes severe constraints on any epistemology - it is historically *the* primary battleground between rationalism and empiricism - but psychology's infection with various forms of logical positivism (unfortunately, still rampant and still virulent; [for an analysis of one example see Bickhard et al., 1985]), with its constituent empiricism, has made it impossible to address or take into account these constraints. In contrast, for Piaget the problem of necessity was absolutely central: 'The fundamental problem of all epistemology, but especially of genetic epistemology, is to understand how the mind attains the construction of necessary relations' [Piaget, 1950, p. 23; quoted in Kitchener, 1986, p. 80]. The centrality of the epistemological problem of necessity in Piaget's thinking attests to his perspicaciousness, especially in the historical midst of the blindness of positivism, but, we will find, Piaget's structuralism precludes a viable solution to this problem.

The path from Piaget's structuralism to his attempted answer to the problem of necessity is complex, and is via several intermediary consequences. I will trace these consequences and some of their interrelationships in Piaget's thinking, and show how they yielded his approach to necessity. I will also show that structuralism per se is fundamentally flawed (in addition to generating several additional unacceptable consequences), and will outline an alternative conception of knowledge that offers a corresponding alternative approach to necessity without the aporias of structuralism.

The discussion will require two fundamental differentiations within Piaget's thought: (1) a distinction between the nature of knowledge and the origins of knowledge, and (2) within the issues concerning the nature of knowledge, a distinction between Piaget's insight into the general operative character of knowledge and his structuralist approach to modeling that character. Both Piaget's constructivism regarding the origins of knowledge and his general operative perspective on the nature of knowledge will be sustained in the offered alternative solution to the epistemological problem of logical necessity (though not in unchanged form); in fact, they are integrated in a new and deeper manner, with constructivism concerning the origins of knowledge shown to be a logical consequence of an operative (interactive) nature of knowledge. Structuralism, however, is shown to be a version of a logically incoherent approach to the nature of knowledge, to be inconsistent with one of Piaget's central arguments for constructivism, and to be at the center of a number of other inadequate and erroneous positions within Piaget's writings - positions concerning epistemology, evolution, and even necessity itself (table I).

Table I. Three views of logical necessity

	Empiricism	Piaget's structuralism	Interactivism
What is known	Actual objects, properties, events, states of affairs	Actual and potential states; potential transformations	Potential interactions
How is it represented	Encoding correspondences	Structural encoding correspondences	Functional differentiation, implicit definition
Origins of representations	Passive 'imitation': transduction, induction	Active imitation; constructivism with necessary endogenous groping; equilibration; phenocopy	Meta-processes of variation and selection; groping heuristics can be constructed
	Combinatorics of encoding elements	Conflates adaptedness and adaptability; no ontological system distinction	Unconfounds
		Infinite regress of origins	Avoids
Action and representation	Action distinct from representation; action may originate with variation and selection	Attempts emergence of representation in terms of coordinations of actions; never explicated	Representation a functional emergence from interactive ability
Nature of necessity	Convention; ad hoc modal encoding	Mathematical closure	Invariance across a space of possibilities
Knowledge of necessity	Ad hoc presence of encodings	Mathematical closure; conflated with nature of necessity	Higher level representation of invariance property of lower level relation; no conflation with nature of necessity
Origins of knowledge of necessity	Aporia	Undifferentiated intrinsic and heuristic constructive tendencies	Differentiated intrinsic and heuristic constructive tendencies

Piaget's Early Stance

In *The Origins of Intelligence in Children* [Piaget, 1952], and in *The Psychology of Intelligence* [Piaget, 1966], Piaget explicitly argues against random trial and error models. His reasoning is interesting, and presages much, though not all, of his later positions. There are two essential parts to this early stance: First, Piaget argues that a trial and error model makes it impossible to account for the accumulation of knowledge over time - knowledge stemming from earlier successes and failures in earlier trials and errors. He considers random trial and error constructions to be trials of context-independent elements that either work or do not work in isolation. In other words, he does not allow for trial and error models with a system of heuristics for trials - heuristics that themselves can develop into more and more powerful heuristics, and for trials that are themselves recursive variations and constructions on the base of already constructed system organizations. He does not allow for trial and error models that contain a mental ontology.

Without such a realm of heuristics and recursive system constructions, Piaget's critique of the impossibility of knowledge accumulation from past successes and failures follows, but why would Piaget ignore such possibilities of accumulation in trial and error models? There seem to be two related reasons: Piaget was addressing an essentially behaviorist version of trial and error in which such internal mental ontologies were forbidden, and he also had in mind neo-Darwinian evolutionary models in which such a realm of 'memories' of previous failures was also considered to be absent - evolutionary variation was unguidedly random and selection was a mere 'filtering'. This general understanding of Piaget's position is consonant with his point that trial and error processes do occur, but they seem to be 'in the margin, or in the vanguard' of intelligent inquiry - 'only when the material exceeds the subject's understanding' - rather than at the 'point of departure' of intelligence [Piaget, 1966, p. 95; see also the discussion beginning on p. 395 in Piaget, 1952]. That is, trial and error processes are not sufficient to account for intelligence, but must be understood to operate with respect to another, more fundamental, realm of cognition and intelligence.

Piaget's second argument points in this same direction. It is a discussion of an additional sense in which context-independent trial and error models cannot be considered to be adequate. The point here is that trial and error processes, when they do occur, are evoked and guided by some prior problem definition, some 'disequilibrium', in already existing, 'previously constructed mechanisms' [Piaget, 1966, p. 96]. That is, problematic disequilibria arise out of prior system organizations; trials at problem solution are not simply random, but are guided by previously constructed heuristics; and trials are not independent elements, but are instead recursive constructions on and with already accumulated knowledge. In the context of the then dominant behaviorism, it is quite correct and perfectly understandable for Piaget to have devoted extensive effort to the establishment of a realm of cognitive ontology - an ontology of problem solving heuristics - that nowadays is generally taken for granted.

Adaptedness and Adaptability

Piaget's discussion of trial and error processes is part of an analysis of the nature of intelligence. His conclusion is that intelligence cannot be accounted for by simple context-independent trial and error [as claimed, for example, by Thorndike: Piaget, 1952, p. 395, 1966, p. 93], since trials and errors can only be understood within the context of an already existing system. This discussion of Piaget's is correct in its general opposition to behaviorist models, but it contains a subtle confusion that seems to have formed part of the conceptual context for Piaget's later stances regarding variation and selection models.

Piaget defines intelligence as the highest form of 'adaptation' [Piaget, 1966, p. 7], and then attempts to understand what adaptation is. He claims that 'adaptation must be described as an equilibrium between the action of the organism on the environment and vice versa' [Piaget, 1966, p. 7]. This immediately introduces the issue of how much equilibrium, or what sort of equilibrium, is required in order to be designated as intelligent - 'it is sufficient that we be agreed on the degree of complexity ... which we shall call "intelligent". But here difficulties arise, since the lower demarcation line remains arbitrary' [Piaget, 1966, p. 9]. In response to this problem of lower bounds, Piaget concludes that 'it is ... possible to define intelligence by the direction towards which its development is turned, without insisting on the question of boundaries' [Piaget, 1966, p. 10].

This directional-tendency view of intelligence yields the point that 'intelligence tends towards reversible mobility' [Piaget, 1966, p. 11]. Because 'reversibility is the very criterion of equilibrium' [Piaget, 1966, p. 11], we find that 'To define intelligence in terms of the progressive reversibility of the mobile structures which it forms is therefore to repeat, in different words, that intelligence constitutes the state of equilibrium towards which tend all the successive adaptations of a sensori-motor and cognitive nature' [Piaget, 1966, p. 11]. Piaget, then, defines intelligence in terms of the (equilibrated, reversible) knowledge (mobile structures) toward which it tends to develop.

The problem with this discussion is that it fails to differentiate, in fact, it explicitly conflates, *adaptedness* and *adaptability* [Simpson, 1967]. *Adaptedness* is a relationship to a particular set of environmental conditions, a particular niche, while *adaptability* is a relationship to potential changes in conditions. Adaptedness concerns the ability of a species to survive and reproduce in particular fixed conditions, while adaptability concerns the ability of a species to survive and reproduce in novel changing conditions. In general, the more adapted to some particular conditions a species is, e.g., anaerobic environments, the less adaptable it is to changes in those conditions. That is, adaptedness and adaptability tend to be negatively related to each other [though not necessarily so: e.g., Bickhard, 1973].

With respect to the individual, this distinction corresponds to a distinction between the ability to function successfully under particular conditions, and the meta-ability to develop new abilities for new conditions. Piaget's discussion attempts to capture the meta-ability of development, but makes that attempt in terms of 'adaptations' rather than adaptability, in terms of the equilibrated structures towards which intelligence tends instead of the meta-processes that manifest that developmental tendency. The definition is given in terms of a directional tendency as a solution to the lower bounds problems, and the lower bounds problem occurs precisely because, for Piaget, adaptability cannot be defined in terms of a separate meta-process, but must instead be defined in terms of adapted process - which immediately yields the question of how much adaptedness is enough?

In the context of a rampant behaviorism, this is not surprising: the fundamental issue is that of the necessity of *any* ontological realm of cognitive structures and processes. The distinction between the interactive flexibilities manifested by goal-directed system organizations, and the flexibility introduced by the meta-ability to *change* system organizations, is a secondary issue. Nevertheless, I wish to suggest that Piaget's implicit conflation of this distinction manifests seeds of later errors and perplexities.

In particular, the view expressed in these early works: (1) sets up a straw man version of trial and error, or variation and selection, as ruling out any ontological realm in which can occur problem definition, heuristic guidance of variation, development of heuristics, and recursive constructive variations - variation and selection models are presumed to be committed to shallow cognitive ontologies, and consequently to purely random, unguidedly random, context-independent variations, and to simple filtering environmental selections; and (2) establishes a view of intelligence, development, and evolution as being defined in terms of the equilibrated systems toward which they tend, and, furthermore, as being evoked and guided by problems, disequilibria, internal to earlier versions of those systems. That is, development and evolution are guided from within the structures that development and evolution are presumed to construct: Here we find in embryonic form the teleonomy that becomes explicit in Piaget's later writings.

The most important problem that will be carried forward from this early stance is that adaptation and adaptability are resident in the same structures, and that there is only one sort of structure for them to be resident in. Structures are both that which is adapted *and* that which engages in adaptation. Consequently, there is no distinction between an ontological level of systems that engage in interaction with the environment and a level of meta-systems that can change those first systems, and there is no ready possibility of introducing such a distinction of ontological levels because there is only one sort of ontology - structures - available in the whole model. In further consequence, there is no possibility of exploring differences *between* the development and evolution of those two levels of system, nor ontological and developmental distinctions and differences *within* either of these levels.

Argument from Design

The internal, teleonomic guidance of evolution and development has become a well developed and extensively argued position by the time of *Biology and Knowledge* [Piaget, 1971b]. The general discussion is in terms of various forms and levels of autoregulations that constitute living beings. In the evolutionary realm, Piaget argues that blind chance variation and selection is simply inadequate to account for the facts of evolution - 'the mutationist and neo-Darwinist traditions ... overestimate ... the possibility of explaining things by chance and [are] satisfied with a simplistic model [of] selection by a sorting out process' [Piaget, 1971b, p. 305]. 'From the point of view of the formation of new combinations, [this neo- Darwinist] solution implies a probabilistic justification, which would be easy enough with minor variations but quite unacceptable for organs such as the eye, etc.' [Piaget, 1971b, p. 292].

Instead, Piaget proposes that, via reverse regulations from the phenotype back to the genome, the genetic system obtains 'information about the results of its activity' [Piaget, 1971b, p. 305]. That is, the genetic system can learn from its successes and failures, can accumulate information, just as Piaget had previously argued for intelligence. In effect, he is here arguing that in evolution as well as in cognition there is an ontology of autoregulatory heuristics and recursive constructions.

Piaget argues that this position does not constitute Lamarckianism because he is not proposing any sense in which the environment simply impresses itself upon the genome. Rather, the phenotype is seen as an active response to the pressures of the environment on the part of the organism, with the results of that response being assimilated at the level of the genome [Piaget, 1971b, p. 289]. Piaget is looking for a way out of what he sees as a dilemma 'between purely endogenous variations (from preformation to haphazard mutation) and selection after the event on the one hand, and environmental influence and automatic hereditary fixation on the other' [Piaget, 1971b, p. 120]. Piaget's concern with an alternative to this dilemma remained a major focus throughout his writings.

It is important to note the form of Piaget's argument here: His first claim is that 'the random variation theory ... [is] ... both mathematically and biologically insufficient to take account of the evolutionary processes' [Piaget, 1971b, p. 175]. His second claim is that reverse regulation from phenotype to genetic system is possible, and could account for the facts that random variation cannot account for. Piaget's argument for this second claim - for the possibility of such genetic assimilation of phenotypic experience - is partly in terms of a suggestive model by which it could occur, and partly in terms of some at best suggestive empirical evidence that it does in fact occur [Piaget, 1971b]. Piaget's deepest argument, however, is his first claim, an in-principle claim that random variations and filtering selections are intrinsically - *mathematically* - insufficient to account for evolution, particularly for such adaptations as the eye. This argument for the insufficiency of randomness is an *argument from design:* such phenomena as the eye are too complex and too perfectly designed for their function for mere randomness to be able to account for them.

Because such arguments from design have had little success in persuading most evolutionary biologists, it is of interest to ask why Piaget felt so compelled by it (arguments from design for the existence of God in fact, were a primary motivating concern for Darwin, and the theory of evolution taken by Darwin, and has been taken by evolutionary biologists since then, as rendering all arguments from design as nugatory) [Bowler, 1984; Gruber, 1981; Mayr, 1982]. I would like to suggest that a fundamental reason is that Piaget had in mind a special and powerful case of argument from design that seemed to withstand all possible counterarguments of neo-Darwinians. This case was in fact a central motivating problem for Piaget's genetic epistemology throughout his career. The problem is to be able to explain the epistemological emergence of logico-mathematical knowledge, and, in particular, of the logical *necessity* of such knowledge. To address it, I must first turn from the biological realm of Piaget's discussion to the cognitive.

Logico-Mathematical Knowledge

The central theme of *Biology and Knowledge* is the relationship between biological processes and organizations and cognitive processes and organizations. Piaget sees the cognitive emerging from the biological as a higher form of autoregulation, with logic and mathematics being the most mobile and reversible, the most equilibrated, forms of such regulations. Logic and mathematics, however, are more than just higher forms of cognition and representation. Logical and mathematical forms of organization are central to ***all*** cognition. 'Representation ... demands ... a logico-mathematical framework outside which there can be no representation at any observable level' [Piaget, 1971b, p · 334], and at the representational level there is a 'necessity for a logico-mathematical framework' [Piaget, 1971b, p. 335]. Furthermore 'logico-mathematical structures fill the same sort of role at the representational level as do hereditary frameworks at the initial learning stages' [Piaget, 1971b, p. 335]. That is, rudimentary forms of such organizations are foundational to and productive of all other realms of representation.

In this respect, the question of the relationship between logico-mathematical organizations and the world is highly parallel to the question of the relationship between the genome and the world, and so also is Piaget's answer**:** Piaget sees logico-mathematical frameworks yielding knowledge of the world and assimilating knowledge from the world in much the same manner - a cognitive version of phenocopy - in which he sees the genome both yielding and assimilating information from the phenotype. This parallelism is recapitulated at higher levels, including the relationship between the formalisms of mathematical physics and theoretical physics [Piaget, 1971b, p. 344].

The Harmony of Mathematics:
Anticipation and Necessity

The argument from design claims that structures such as the eye are too complex and perfectly suited to their function in the world to be explained by mere chance. The harmony between mathematics and the real world is the special case of perfect design that seems to be ultimately persuasive to Piaget.

There are two parts to this harmony: one is the sense in which mathematics over and over again anticipates organizations in the world, and the other is the logical necessity of mathematics. Piaget's concern with mathematical anticipations is evidenced in such statements as 'even more striking and more common is the construction of purely abstract mathematical structures, which afterward serve as indispensable frameworks for physical phenomena, without having been intended as such beforehand' [Piaget, 1971b, p. 341]. The argument-from-design consideration is explicit in 'That random mutations and Darwinian selection should explain how hooves and fins come to be formed is conceivable, strictly speaking (although I myself do not believe it); but to explain on the basis of this model why Riemann's work on abstraction should have acquired a meaning in physics thanks to Einstein - that is endowing chance with remarkable intelligence and turning selection into an intentional choice capable of influencing, in a rather frightening way, the part that is still behind the scenes' [Piaget, 1971b, p. 342]. In Piaget's view, chance cannot produce such perfect anticipations.

Concerning the logical necessity of mathematics, we find '[logico-mathematical] structures cannot be caused by mere hereditary transmission, for if [they were] they would be neither necessary nor general' [Piaget, 1971b, p. 322]; 'if [hereditarianism] is right, mathematics loses all its "necessity", since a hereditary characteristic is no more than what it is' [Piaget, 1971b, p. 325]; and, with great explicitness, 'it is unthinkable that the human brain's capacity for constructing logico-mathematical structures that are so admirably adapted to physical reality should be explained away be mere selection, as the mutationists do, for factors of utility and survival would have led only to intellectual instruments of a crudely approximate kind, loosely sufficient for the life of the species and its individual members, and never to that precision and, above all, that intrinsic necessity which demand a much more penetrating explanation of adaptation [than] by a posteriori selection within random variation' [Piaget, 1971b, p. 274]. In Piaget's view, chance cannot produce necessity.

Two Perplexities

Piaget devotes much of *Biology and Knowledge* to making the case that the genome is capable of accumulating knowledge, and then bases his discussion of cognition in general, and mathematics in particular, on derivations from this biological foundation. There are at least two points of perplexity in this argument, however. The first is that, whatever the merits of the case for genomic phenocopy, the possibility of such accumulation of knowledge at the level of cognition has not been at issue for a very long time, so why was Piaget so concerned about it? That is, was Piaget's concern with evolutionary phenocopy only a *parallel* concern to that with individual epistemology, or is there a deeper logical connection between the two?

The second perplexity is that, however much accumulation of knowledge from 'memory' of previous successes and failures there may be, the outer envelope, the margin, 'vanguard', or leading edge of variation and environmental selection, of development and evolution, might still be claimed to be essentially random: No matter how sophisticated the posing of problems to the environment by the organism, no matter how complex the heuristics for trials at problem solutions, and no matter how elaborate the foundation for further constructions, if those posings and heuristics and foundations are themselves ultimately the *product* of random variation and selection, then Piaget's concerns about the in-principle inadequacy of randomness are not helped by his argument at all. That is, if the ontological realm for the accumulation of knowledge, whether at the biological or cognitive level, is itself ultimately due to random variation and selection, then that is simply a demonstration of the sophisticated power of randomness, rather than an alternative to randomness as Piaget wanted it to be.

A Non Sequitur

The existence of a realm of heuristic knowledge accumulation in the service of further knowledge construction is the focus of the argument that Piaget presents, but he concludes more than that. Such a realm is at best controversial at the level of biology, but is taken for granted (contemporarily) at the cognitive level. The problem for Piaget is that even granting such a realm does not resolve the deepest issues that Piaget was concerned with. In particular, however much such internal active accumulation of knowledge may seem to make such grand designs as the eye seem more plausible, an ultimate randomness at the leading edge of such a realm would leave Piaget's in-principle questions about the harmony of mathematics with the world - the anticipations and logical necessity - untouched. Randomness, no matter how mediated by heuristic accumulations from past random successes and failures, would still seem to be unable per se to account for mathematical anticipations and logical necessity - there must be some other source of such harmony.

Piaget is partially aware of this issue, and his answer to it is in part direct and in part contained in some implicit presuppositions.

The direct part of his answer is that equilibration via autoregulations is itself a source such harmony - both anticipations and necessity. 'The necessary character of logico-mathematical structures, then, does not in the least prove them to be hereditary but emerges from their progressive equilibration by dint of autoregulation' [Piaget, 1971b, p. 317]. Piaget's solution to the problem of the anticipatory harmony between mathematics and the world is not as explicitly presented, but is suggested in 'knowledge of environment and of objects which is so admirably attained by the human mind is only so attained by virtue of an extension of the organization's structures into the universe as a whole.... physical knowledge is an assimilation of the real world into logico-mathematical structures [since] the organization belonging to a subject or to any living creature is a condition of exchanges with [the] environment' [Piaget, 1971b, p. 338f.]. Further 'there is no organizing function, at whatever level, that does not harmonize with the environment; the harmony between mathematics and experience is just one example of this' [Piaget, 1971b, p. 345]. In other words, logico-mathematical structures are the necessary frameworks for all knowledge, since they are 'simply' the abstracted forms of such knowledge: 'any kind of knowledge about an object is always an assimilation into schemes, and these schemes contain an organization, however elementary, which may be logical or mathematical' [Piaget, 1971b, p. 335] and, less equivocally, 'physical and experimental knowledge cannot possibly be established without some structuration and logico-mathematical framework' [Piaget, 1971b, p. 342].

Therefore, when such mathematical structures are taken as domains of study and development in their own right, it is perfectly reasonable that independent mathematical developments might anticipate their applications, as frameworks, to the world.

Logical Necessity

Although Piaget's answer to the problem of the anticipatory harmony between mathematics and the world is less explicitly stated than is his answer to the problem of logical necessity, it is nevertheless a more satisfactory answer. Piaget addresses the issue of anticipation by arguing that mathematical structures are necessary frameworks for understanding the world. A new mathematical development constitutes a new potential such framework, so it should not be surprising if such a potential framework should become actually used. His explicit answer to the problem of logical necessity, however, is that equilibration is itself the source of necessity. Understanding what Piaget intends by this claim requires a deeper analysis of Piaget's conception of the nature of knowledge and of necessity.

First, Piaget held that there are two primitive aspects of knowledge: knowledge of states - the *figurative* aspect - and knowledge of transformations - the *operative* aspect [Piaget, 1970, p. 14, 1977, p. 18]. Both figurative and operative aspects involve knowledge of *potentialities:* of potential states, or of potential transformations among those states. This move to potentiality as the fundamental focus of knowledge - instead of knowledge only of actualities (e.g., things or facts) in the world - is one of Piaget's major insights, and one that most of the rest of psychology, e.g., cognitive science, has not yet caught up to.

Second, knowledge of potentialities is constituted in systems that correspond to those potentialities, that are isomorphic to them. 'Knowing reality means constructing systems of transformations that correspond, more or less adequately, to reality. They are more or less isomorphic to transformations of reality' [Piaget, 1970, p. 15]. This point is intrinsic in Piaget's structuralism, both early and late - structures are in correspondence with, are isomorphic to, what they 'know' or represent. The point is made explicitly in many places in Piaget's writings, but is most often presupposed rather than stated or addressed, and it is nowhere elaborated or defended. A clear example of such a presupposition even in Piaget's later works is the discussion of the removal and reinsertion of single operations in a structure [Inhelder and Piaget, 1980, p. 22]; another would be the similar individualization of 'possibilities' in their acquisition during the development of possibility and necessity [Piaget, 1987]. This presupposition of the morphic correspondence character of structuralism is most often implicit rather than addressed because it does not lie on the line of any of Piaget's major concerns - he did not understand it to be problematic, and so neither defended it nor developed any alternative to it. His main battles were elsewhere. This presupposition, however, will be central to my analysis and criticism of some of the positions that Piaget did explicitly develop.

Third, this realm of potential transformations among states as a focus of knowledge gives rise to a second such realm - a realm of the potential *coordinations* among those potential transformations, a realm of *meta*-potentialities. Knowledge of these meta-potentialities of coordinations is at first piecemeal, one or a few at a time, but becomes progressively more organized and integrated with development. '... all knowledge is linked to action and... the evolution of actions presupposes coordination' [Piaget, 1971b, p 28]. 'First of all, there is a beginning of coordination between segments of overt behavior ... finally, they take the form of mobile and reversible operational structures' [Inhelder and Piaget, 1964, p. 291].

Fourth, coordinations which are known but which are not yet integrated into reversible structures are called *regulations,* while coordinations which *are* integrated into such structures are called *operations* [Inhelder and Piaget, 1958, p. 246]. Operations, then, are coordinations that are embedded in integrated structures.

Fifth, the critical characteristic of operational structures that differentiates them from regulatory knowledge is that operational knowledge is 'reversible' - capable of reverse transformations - while regulatory knowledge is not. 'Reversibility is defined as the permanent possibility of returning to the starting point of the operation in question' [Inhelder and Piaget, 1958, p. 272]. 'The difference between these two sorts of mechanisms [regulations and operations] is that reversibility remains incomplete in the first case but is achieved in the second' [Inhelder and Piaget, 1958, p. 246]. 'The characteristic of intelligence is not to contemplate but to "transform" and its mechanism is essentially operatory. Operations consist of interiorized and coordinated actions in group structures such as reversibility' [Piaget, 1971a, p. 67].

Sixth, the reason that reversibility is such a critical characteristic is that only with the reversible integration of coordinations is the system able to fully *compensate* for changes - to enact a transformation back to the origin, to reverse a change - and, thus, to maintain its own *stability.* '... stability is the outcome of compensatory forces (which in turn express the reversibility of ... operations). ... stability is a function of compensatory operations, and the reversibility merely expresses their compensatory character' [Inhelder and Piaget, 1964, p. 292].

Seventh, such reversibility, and, thus, full compensation and stability, is a property of *closure* of the structure of operations with respect to the full space of potential ordinations - there must be no potential transformations for which there are no coordinated reversing, compensating, potential transformations in the system. All potential transformations in the structure must stay within the reversibly coordinated domain of the structure - the structure must be closed with respect to its integrated potentialities of transformation and coordination - otherwise, there will be some potential transformation, some potential change, for which no compensation is available. '...the system of exchanges with environment ... in no way excludes a closure, in the sense of a cyclic rather than a lineal order [of the exchanges]' [Piaget, 1971b, p. 155]. '... the structure becomes whole and closed; that is, relations within it are interdependent and can be composed among themselves without recourse to anything outside the system' [Piaget, 1971b, p. 316].

Eighth, there are certain general forms of such closed structural coordinations of operations. 'Operations are a continuation of actions; they express certain forms of co-ordination which are general to all actions; whether or not the co-ordination is complete, operations and pre-operational co-ordinations enter into the most diverse kinds of behaviour' [Inhelder and Piaget, 1964, p. 291] These general forms will be the logico-mathematical structures of the groupings, lattices, groups, and so on.

Ninth, it is precisely such structural closure that yields the emergence of *logical necessity* [Piaget, 1971b, p. 3l6f.J. '...the ... subject ... accedes to logical necessity at ... the level of the closure of operational structures' [Piaget, 1971b, p. 322]. *... inferential necessity is an index of the closure of an operatory structure' [Piaget 1985. p. 99). And, tying several points together. 'In their final operatory form, [logico-mathematical] structures attain complete compensation* [Piaget, 1985, p. 133].

Tenth, such properties of closure, reversibility, and compensation *constitute* the property of *equilibrium.* Full equilibrium is precisely the ability to fully compensate, which requires full reversibility, which requires operational closure - which, in turn, yields the emergence of logical necessity. ... a system is in equilibrium when a perturbation which modifies the state of the system has its counterpart in a spontaneous action which compensates it' [Inhelder and Piaget, 1958, p. 243]. '... equilibrium in action is defined as an active compensation set up by the subject against exterior disturbances, whether experienced or anticipated' [Piaget, 1971b, p. 12]. 'Operational equilibrium ... is essentially characterized by reversibility ... or, more particularly, by a stable form of compensation systems' [Piaget, 1971b, p. 25]. Note that equilibrium is a relationship between the organism and its environment, between structures and the environment, *where the environment is conceptualized in terms of both transformational potentialities and coordinative meta-potentialities.* Equilibrium is not just a property *within* an organism that is maintained in the face of actual environmental disturbances and selection pressures - this follows from the point that knowledge is fundamentally of the environment as potentiality and meta-potentiality, not just as actuality. Regulatory 'equilibrium' is the readiness for coordinative compensation against potential disturbances, and *perfect* equilibrium, structural or logico-mathematical equilibrium, is the readiness for coordinative compensation against *all possible* such disturbances within some domain of interaction. Logico-mathematical structures are closed; they exhaust some domain of coordinative meta-potentialities by containing all the potential coordinations in that domain, and thus every possible path has a return.

Eleventh, and finally, *equilibration* is precisely the process of reequilibration in the face of disturbances (actual or anticipated) - of the construction of compensatory coordinations - and, consequently, of the progressive construction of more and more equilibrated organizations of coordinations, of higher and higher forms of equilibrium. Equilibration 'amounts ... to putting up active compensations against outside perturbations ... Obviously, then, there is a continuity between the equilibrium attained and the process of equilibration itself [Piaget, 1971b, p. 25]. Furthermore, 'operations of thought and especially those of ... logico- mathematical thought ... [have as] their prime cause ... a factor of gradual equilibration' [Piaget, 1971b, p. 1lf.], and 'logico- mathematical structures... attain permanent equilibrium despite the constantly renewed constructions which characterize their own evolution' [Piaget, 1971b, p. 356].

We are now in a position to return to the question of why Piaget claimed that equilibration itself yields necessity. Within the theoretical organization just outlined, the answer becomes obvious: equilibration is the process of the construction of progressively more closed organizations of coordination and, at the point where that closure is attained - at the point where those organizations become operational structures - necessity emerges. At the point when the structure exhaustively captures the (relevant) metapotentialities of coordinations - *contains* all of them, and thus is closed with respect to them - the structure *attains* logico-mathematical necessity.

Unfortunately, this only addresses a part of the issue. It explains how, in Piaget's view, 'equilibration is supposed to *functionally* produce necessity via the attainment of closure, but, in Piaget's own analysis, if the *process* of equilibration - or the process that *produces* equilibration - were one of mere random variation and selection, if structures were formed by mere random variation and selection, then the structures could be expected to have at best 'crudely approximate' organization, sufficient for utility and survival, but not to have the precision and necessity that we in fact find:

> Biological selection is, in fact, related to survival, whereas the victory of one idea over another depends, in the final analysis, on the value of the truth contained in it. ... It is unthinkable that the human brain's capacity for constructing logico-mathematical structures that are so admirably adapted to physical reality should be explained away by mere selection, as the mutationists do, for factors of utility and survival would have led only to intellectual instruments of a crudely approximate kind, loosely sufficient for the life of the species and its individual members, and never to that precision and, above all, that intrinsic necessity which demand a much more penetrating explanation [than] adaptation by a posteriori selection within random variations. We can, then, attach no credence to the mutationist solution [Piaget, 1971b, p. 274f.].

Again we encounter Piaget's claim that equilibration - or some property of it - is itself the source of such precision and necessity - a source that transcends mere randomness - but how and why should that be (necessarily) so? One perspective on this point is contained in

> the whole problem is to know whether this influence [from the world to the mathematical frames for understanding the world - in this immediate discussion, from experimental physics to mathematical physics] is psychological, that is, relating to the choice of problems and the interests dictated by these choices, or whether it is epistemological - that is, including a transfer of truth. ...
>
> Any biologist reading this summary of analyses is bound to think of situations in which phenotypic variation precedes the appearance of a genotype that seems to be an imitation of it, which is sometimes called a phenocopy precisely in order to show that an active and endogenous imitation has taken place [Piaget, 1971b, p. 344].

Piaget is at pains to point out that

> To the extent that the correspondence [between mathematical structures and physical data] is successful, the mathematician still achieves nothing by [merely] 'imitating' the physical data by means of his abstract structures; it is only by means of internal and endogenous recombinations that he can reach those data, borrowing nothing from the external 'representations' which he integrates and reconstructs with full autonomy [Piaget, 1971b, p. 344].

His concern here is to emphasize the necessary active reconstructions of the mathematical physicist, rather than some relatively passive imitation of the data. This is also why he speaks of phenocopy as 'active and endogenous imitation' rather than just as imitation: the concern here is precisely the cognitive version of his earlier argument that his position does not constitute Lamarckianism at the level of biological evolution because the organism, the genotype, is an active constructor rather than a passive receiver of impressions.

Activity notwithstanding, the aspect of this discussion that I wish to emphasize is that 'phenocopy', whether of the biological or cognitive variety - that epistemological influence - involves a 'transfer of truth' into the system, that it involves a relationship of imitation, however actively arrived at. Piaget's discussion suggests that this is the fundamental reason that mathematics has logical necessity: mathematical structures transfer truth, they import truth, from the world (from the meta-potentialities of coordinations of transformations with the world), they do not just have the approximative utility of fitting random selection pressures from the world. This presumption of an importation of truth, an active imitation, by a process of equilibration via endogenous autoregulation - a cognitive 'phenocopy' model - is essential to Piaget's answer to the problem of the logical necessity of mathematics.

How Does Truth Get Imported?

The problem is that this 'importation of truth' property of the endogenously active genome or epistemic agent is a non sequitur from Piaget's previous arguments. An ontological realm, whether biological or cognitive, which actively poses problems, heuristically proposes solutions, and accumulates information from past successes and failures - 'it is obvious that all learning by trial and error (or groping) presupposes feedback structures of such a kind that the lesson learned by each trial has a chain reaction on those that follow' [Piaget, 1971b, p. 11] - does not automatically accomplish anything like a 'transfer of truth' from the world (of actualities *or* potentialities) into the system. Piaget does not address the issue of how such processes *could* accomplish such an importation, nor whether such importations are possible at all, nor even what 'truth importation' would mean. It is presupposed without analysis, but does not follow from any previous argument. Why such a non sequitur in Piaget's thinking?

Structuralism and Constructivism - Gathering Loose Ends

At this point it becomes possible to start pulling together several loose ends that have developed in this discussion: why the non sequitur of truth importation as a property of active equilibration; why the perplexing concern with the active assimilations of the *genome* when such properties are already accepted for cognition; and why and with what consequences the conflation between adaptedness and adaptability? Clearly, I intend to suggest some relationships among these.

Piaget was throughout his career greatly concerned with the necessarily active and constructive character of knowledge, and devoted to an ongoing criticism of models of passive knowledge acquisition, whether in the cognitive version of copy theories [Piaget, 1971b, p. 361] or in the biological version of Lamarckianism [Piaget, 1971b, p. 289], which Piaget equates with epistemological empiricism. Together with this fundamentally constructivist conception of the acquisition of knowledge, however, Piaget held to a deeply structuralist conception of the nature of knowledge. That is, knowledge was acquired only via active constructions, and was impossible to attain via passive copies, but the nature of what was attained was structures that 'imitated', or 'corresponded with', the environment. In other words, my claim is that Piaget's arguments. were devoted to the establishment of the necessarily active constructivist character of the *acquisition* of knowledge, and that the tacked-on presupposition of 'importation of truth' or 'correspondence' as an aspect of this process derives from presuppositions concerning the structural *nature* of knowledge.

This distinction between Piaget's assumptions concerning the acquisition of knowledge and those concerning the nature of knowledge begins to yield an understanding of the previous loose ends. Insofar as the acquisition of knowledge *must* be active and constructive, and insofar as the *genome* is regarded as an epistemological locus [Cellerier, 1984, argues that Piaget would be on stronger grounds here if he were to take the gene pool as the relevant locus], then the *genome* must be epistemologically active and constructive, and *something* like phenocopy *must* be true. Conversely, if the epistemic process of evolution were 'merely' a matter of random variation and selection without active groping construction, then what would be the logical necessity of positing epistemic active construction on the part of human beings? If random variation and selection sufficed at the evolutionary epistemic level, why not for human beings too? These points attain special force when logico-mathematical structures, with their anticipatory and logical-necessity harmony, are taken into account: such structures are both individual *and* biological epistemic accomplishments, and the problem of their origin, the possibility of their emergence, must be addressed at both levels. My suggestion, then, is in part that Piaget's concern with biological issues of an active and constructivist genome stems from his concern for a general constructivist view of the origin of knowledge – both the developmental and the evolutionary origin.

This point must be amplified, however. If constructions per se as the source of potential new knowledge were the only issue, then *random* constructive variations and environmental filtering selections would seem to suffice, at least in principle, and at least a version of neo-Darwinianism would therefore suffice, but clearly Piaget would be greatly dissatisfied with this. Such a model would involve necessary constructions, on both the biological and the individual levels, but the guiding, teleonomic constructive heuristics would be missing at the biological level, and, in Piaget's view, the filtering selections applied to merely *random* constructions would produce at best approximative utilitarian solutions, rather than mathematical structures, at both levels. Piaget needs the structuralist correspondences at both levels, because that is the nature of knowledge at both levels, and most especially so in the case of mathematical knowledge, and he seems to feel that is only obtained by a truly active and guiding constructive epistemic agent: the intuition, though not developed, seems to be that only via endogenously active explorations will structures of potential transformations or coordinations be 'perfectly filled out', perfectly imitated, by the system variations - be 'filled out' from the inside, so to speak - while, with merely random variations, structures of potential transformations will be filled out' in at best a hit and miss manner that will approximately capture the structure of potentiality, but will not truly 'imitate' or transfer' that structure into the system. In this case of the meta-potentialities of coordinations, this would mean that exhaustive closure would not occur, and, therefore, necessity would never emerge. The 'crudely approximative' capturing of some or many coordinations by unguided random variations might suffice for some or many practical purposes, but logical necessity *only* emerges when the *entire* domain of metapotential coordinations, without exception, is structurally captured, thus attaining closure. Therefore he must have such an active and guiding constructive and assimilative 'agent', or system, at both the individual and the biological levels, and, therefore, we find the postulation of, and the extensive argumentation for, phenocopy at the level of biology.

To recapitulate: Structuralism yields a model of logico-mathematical knowledge as being perfectly equilibrated, 'truth importing' structures - structures with perfect correspondence to the meta-potentialities of coordinations with respect to the environment. Such perfect structures cannot be considered to be produced by mere random variations and filtering selections, because these would at best produce coarse and crude approximative structures, not the perfect anticipative and logical-necessity harmony of logico-mathematical knowledge. Therefore, endogenously active heuristic constructions are necessary.

Once again, however, if such endogenously active heuristics were *themselves* the products of randomness, Piaget's in-principle argument would fail. Piaget denies that that could be so [Piattelli-Palmarini, 1980, p. 281], but, more fundamentally, it is impossible to model or even to seriously consider how (or that) it might be so within Piaget's framework: to do so would require (1) a model of random variation and selection processes; (2) a model of endogenous constructive heuristics developmentally constructed by those variations, and (3) a model of systems for interacting with the world constructed by those heuristics - and Piaget cannot begin to make such distinctions within his single mental ontological level of structures.

In other words, at this point, Piaget's conflation of adaptedness and adaptability again comes into view. There are two parts to this conflation. First, Piaget sees the evocation and guidance of the development and evolution of intelligence as stemming from *inside* the system, whether the individual or the genome, rather than from outside either in the form of passive impressed empiricist copies or in the form of random filtering selections. To posit an interactive process between the organism and the environment, and a logically separable meta-process that alters and constructs that lower level process via random variations and selections, would seem to eliminate any possibility of such internal evocation and guidance of the development of the system: System development would appear to be solely in the hands of the (random) meta-process, so where could the guidance come from? (I will argue later that this appearance is in error.)

Furthermore - for the second part of the conflation - in Piaget's view it is *only* with such internal evocation and guidance that anything more than simple approximative utilitarian procedures, that anything like logico-mathematical structures, could possibly develop. It is only with such actively guided construction that (perfectly designed) structuralist correspondence or isomorphism will occur. Therefore, from Piaget's structuralist perspective, importation of truth is necessary to explain anything like mathematics, and importation of truth is impossible except for an active *guiding* epistemic system, and, therefore, the constructions and developments of adaptability must be evoked and *guided* from *within* the (partially) adapted system - and, therefore, with only one level of system, *adaptability* is a *resultant* of systems of *adaptedness,* and the strong distinction between adaptability and adaptedness is misguided and misleading.

Historically, of course, this may have worked the other way around: Piaget's deep sense of the necessity of the endogenous constructive activity of the epistemic agent may have in itself required him to defend an appropriate ontological realm for such activity in the face of behaviorists who would have denied it; yielded a subordination of trial and error changes-to-system to the internal guidance of the system itself - of adaptability to adaptedness; and led to a model of such activity at the level of the epistemic genome as well as that of the person. Such a motivational organization of Piaget's conclusions distracts from the logical connections explicated in the previous discussion, and would explain why those logical connections are often relatively undeveloped in Piaget's writings. It also, correspondingly, obscures the role of structuralism as a common presupposition to those logical connections.

A convergent perspective derives from the realization that Piaget's central concern in these discussions was not at all with the structuralist correspondence presuppositions that I have been focusing on. Most commonly, with regard to necessity, it was that by explicating the emergence of necessity in terms of the intrinsic, internal property of the *closure* of a (meta-potential) domain of coordinations, which are intrinsically *relational* between the system and the environment - not resident in either, Piaget had explicated a source of knowledge (of necessity) that was neither environmentally empiricist or Lamarckian, nor preformationistically innate. Necessity - closure - emerged *intrinsically* in the nature of operational equilibration and equilibrium. Again, the correspondence presuppositions of structuralism are of little concern, and are correspondingly presupposed (e.g., in the very notion of capturing *individuated* coordinations until closure is attained) rather than addressed.

Another major theme in Piaget's writings that is involved here, and again, in which the structuralist correspondence assumption is implicit, is his sense of the essential and fundamental involvement of *modality* in knowledge and development - a deep insight so unlike, and so much more perspicacious than, any version of empiricism. The 'truths' that are 'transferred' are truths of *possibilities* - potentialities and meta-potentialities - of actions, transformations, coordinations, and structures of such. Each new equilibratory construction of a new transformation or coordination is an 'opening' of the system to a new possibility (note the individualization), while it is in the 'closure' of such possibilities that necessity emerges, and development - equilibration - may be viewed in a fundamental way as a kind of dialectic between these [see, for example, Piaget, 1969, p. 250; Inhelder and Piaget, 1958, pp. 255-266; Piaget, 1977/1986; Piaget, 1981/1987].

Another, also convergent, aspect of Piaget's motivations, however, highlights the role of structuralism, and provides a deeper positive motivation for Piaget's having restricted himself to only one ontological level of system process, though it still leaves the logical connections and consequences obscure. This motivation was the pervasive analogy that Piaget drew between his epistemological concerns and those of biology. In particular, in analogy to the distinction in biology between structures and their functioning - with the corresponding domains of study being anatomy and physiology, respectively - Piaget posited epistemic structures whose functioning was equilibration [Kitchener, 1986, p. 150]. Again, we end up with only one level of system and process.

Unfortunately, however arrived at, this set of conclusions has put Piaget in at least two untenable positions. The first is the empirical problem that evolution just does not seem to work the way Piaget needs it to: any evidence for anything like phenocopy is at best minimal and subject to alternative interpretations [Simpson, 1967; Piattelli-Palmarini, 1980].

An Infinite Regress of Origins

The second is the deeper, logical problem that, by emphasizing the in-principle *necessity* of the *active* constructive agent in order for epistemic development to occur, Piaget seems to have put himself in a position in which he needs to posit the existence of epistemic agents in order to explain the existence of epistemic agents. That is, Piaget's model of epistemic development would seem to preclude the very possibility of the emergence of epistemic agents - prior to such systems, there would be no locus of such constructive guidance as Piaget posits is necessary for the development of such systems.

Piaget might conceivably counter that the *emergence* of such systems is a different issue than their development once emerged, and that only the latter requires the active endogenous constructions. If, however, their emergence were in turn explicable in terms of mere random variation and filtering selection, then the logical foundations of most of

Piaget's argument would fall: if epistemic guidance and teleonomy could themselves be produced by random variation and selection, then we (again) simply have a model attesting to the power of such randomness, and active guidance is no longer epistemically necessary. Piaget, however, holds, rightly, that life is intrinsically epistemic: life is a matter of certain forms of self-maintaining autoregulation ['Life is essentially autoregulation', Piaget, 1971b, p. 26], and autoregulation is intrinsically epistemic ['knowledge is ... a system of real interactions reflecting the autoregulatory organization of life' Piaget, 1971b, p. 27]. The basic point here is simply that autoregulation requires compensations for and anticipations of the transformational changes in the environment, and such an ability to compensate and anticipate *constitutes* knowledge. But, if the existence of living systems is intrinsically epistemic, then the trap concerning the origins of epistemic systems is complete: epistemic development requires epistemic guidance and construction, but even the most elementary epistemic (living) system necessarily contains epistemic content, which by hypothesis could not have developed without the guidance of an epistemic system, and, therefore, every epistemic system requires a *prior* epistemic system in order to explain its coming into being.

To this it might be countered that Piaget was not attempting, and need not attempt, a model of the ultimate origins of epistemic systems, and, thus, that the criticism misses its mark. But Piaget need not make such an attempt for the criticism to hold. It is an in-principle criticism that *any* account of origins is logically impossible within Piaget's system, and, thus, that the very existence of epistemic systems is a counterexample to Piaget since, by the necessary infinite regress above, that existence should be impossible. The argument is identical in general form to the in-principle argument that Piaget mounts against Chomsky and Fodor's innatism arguments [Piattelli-Palmarini, 1980] - they require that we already have representations in order to be able to get representations, and thus enter into their own infinite regress - and, I will suggest later, there is a mistaken assumption in common between the two positions that underlies both regresses.

The answer to the first perplexity mentioned above - why does Piaget focus so strongly on phenocopy at the biological level when the equivalent is readily granted at the psychological level - was that Piaget viewed such active assimilation from experience to be *epistemologically* essential, and, therefore, essential at the biological level just as much as at the psychological level. Piaget's general stance regarding epistemology, then, *required* that he argue for something like phenocopy in evolution.

The answer to the second perplexity mentioned above - how does Piaget avoid the power of purely random variation and selection if his ontological realm of heuristics and active constructions is itself **a** product of random variation and selection - is somewhat different. Simply put, Piaget did not avoid it, or, more carefully, he avoided it only by positing the logical necessity of active epistemic guidance for the construction of epistemic content - since such endogenous guidance is necessary, merely random variations will not suffice; unfortunately, he thereby committed himself to the impossibility of the emergence of any epistemic system because of the vicious infinite regress of the origin of such epistemic guidance.

Later Developments

The claim here is that Piaget was caught in conceptual trap, not that he was cognizant of that trap. In fact, his thinking continued to develop within the problematic framework I outlined above, and the consequences of that framework became even more explicit. In 1975, in the debate between Piaget and Chomsky, the centrality of the problem of logical necessity is still very explicit - 'The central problem, then is to understand how such [epistemic] operations come about, and why, even though they result from nonpredetermined constructions, they eventually become logically necessary'[Piattelli-Palmarini, 1980, p. 23]. Similarly explicit is the claim that such necessity requires a teleonomy – 'I absolutely refuse, for my part, to think that logico-mathematical structures would owe their origin to chance; there is nothing fortuitous about them. These structures could not be formed by survival selection but by an exact and detailed adaptation to reality' [Piattelli-Palmarini, 1980, p. 59], followed later by the elaboration, 'There are two types of adaptation that have to be distinguished: (1) survival-adaptation, which favors the rate of multiplication and the preservation of the species by the screening of useful and noxious variations, both having occurred prior to this screening and independent of it; and (2) adequacy-adaptation, which implies a teleonomy in relation to the environment... Now, it is this teleonomy that has to be explained because... it could [not] result from a mere sorting' [Piattelli-Palmarini, 1980, p. 281]. As before, Piaget explains such teleonomy in terms of phenocopy [Piattelli-Palmarini, to, p. 59].

Piaget is still critically aware of the importance of the issue of logical necessity, but he still treats it as simply another example of argument from design - he even argues that mere survival selection could not possibly explain something so exquisite as a swallow's refined nest! Even in this case, teleonomy is required to explain how it could have come about [Piattelli-Palmarini, 1980, p. 281]. In this rather incredible assimilation of the issue of logical necessity to the refined complexities of birds' nests, Piaget is drastically weakening his position. The argument from design for such things as birds' nests and eyes depends on an essentially probabilistic argument that the production of such complexity and perfectly refined design is simply too improbable given a random variation and selection model. The argument, in other words, is not one of impossibility in principle, but rather implausibility in consideration of the small probabilities supposedly involved. The strongest version of the argument from design for logical necessity, in contrast, is not a probabilistic argument at all, but, instead, an in-principle argument concerning ontological emergence. Basically, the argument that Piaget limns is not just that the likelihood of chance producing mathematical structures is vanishingly small, but, rather, that random chance is *intrinsically incapable* of producing logical necessity. Not just that the probability is low, but that such a production of necessity by randomness is ontologically impossible. Piaget suggests this argument, but then deflates it by assimilating it to such things as birds' nests.

The argument from design for birds' nests, eyes, and other such products of evolution stems not from any special ontology involved, but rather from their complexity or exquisiteness of functional fit. No ontological emergence is involved. To be persuaded by such arguments in the face of current knowledge concerning selective shaping and progressive evolution seems odd to the point of bizarreness, and Piaget is roundly criticized for his stance during the debate with Chomsky [Piattelli-Palmarini, 1980].

But Piaget is perfectly correct that the epistemological problem of the emergence of logical necessity has essentially not been addressed by biologists other than Piaget himself. Perhaps part of his motivation was that, if teleonomy is required, if adequacy-adaptation is required, to explain logical necessity, then it would also be available to help explain birds' nests. But the strength of the point about necessity depends precisely upon the fact that it is not simply another version of an argument from complexity and functional fit, but is instead an argument about ontological emergence, and Piaget obscures that.

On the other hand, there is an important sense in which Piaget's *theory* obscures the distinction between the ontological emergence of necessity and the origin of exquisitely designed complexity, and perhaps Piaget is himself rendering the issue of ontological emergence as instead 'simply' an issue of the origin of exquisite complexity - is reducing an issue of emergence to an issue of probability - in spite of some passages that seem at least to honor, if not to make, that deeper distinction. This, of course, would reintroduce the question of why Piaget was persuaded by a 'mere' argument-from-design without any distinct supporting argument- from-emergence, especially considering that the appearance of design was precisely one of the major issues to which the theory of evolution was addressed in the first place.

The sense in which Piaget's theory conflates emergence with exquisiteness is that Piaget explicated the growth of knowledge as the construction of structures that progressively corresponded to more and more of the potential transformations and operations in the world, and he explicated necessity as 'emerging' when that filling-out of operational potentialities with correspondent structures reached structures of local saturation or closure, in which all possible operations were already captured, imported, compensatable. Such transformational or operational closure involves potential transformations/operations in all directions among all possibilities, and, thus, implies the reversibility and complete mobility that Piaget so often emphasized, and, further, constitutes a direct model of the algebraic transformation groups that Piaget was so fond of for modeling such structures (i.e., such a saturated organization of transformations, understood as a set of automorphisms, would tend to intrinsically have the properties of associativity, identity, and inverses that would constitute them as a group under composition). In this view, necessity 'emerges' when a condition of saturated importation of truth is reached - a saturation that yields closure, complete mobility, perfect and permanent equilibrium (further accommodations are precluded because the space of possible operations to be accommodated to is already fully captured, is already saturated), and so on. Such saturated closure will *not* obtain if *any* of the possible operations are missed - not imported, not actively imitated. Thus, the 'emergence' of necessity will depend upon *totally* capturing such structures of operational potentialities, not just some partial or ragged substructures produced by the mere random constructions of survival adaptation, and, therefore, endogenous active exploration of those (meta-)potentialities is required. In this perspective, then, the *emergence* of necessity *is* reduced to the *exquisite perfection* of totally capturing the space of potential operations, and the argument for the necessity of endogenous teleonomic activity *is* reduced to the implicit claim that the probability that mere random construction would fully capture such a space of potential operations is vanishingly small. In this view, the ontology of necessity is reduced to the exquisiteness of total, closed structural correspondence, and the argument from emergence is reduced to an argument from probability.

Piaget was never very clear about exactly these points, so it is correspondingly not clear how much this exact reasoning influenced him, but it does follow from other parts of Piaget's position, and it introduces a strong at least motivational coherence into some of his otherwise puzzling positions.

The explication of necessity involved here, however, is in fact seriously deficient: the connection between closure, reversibility, etc. and necessity is not clear and is never well developed [Campbell and Bickhard, 1986]. For example, closed mathematical structures may *have* necessary *properties,* but it does not follow that closed mathematical structures have anything necessarily to do with the *epistemology,* with *knowledge,* of the *property of necessity.* Piaget, in fact, began moving away from this purely structural explication of necessity later in his career, but did not have the opportunity to explore the far reaching ramifications of such a change for the rest of his theory [Campbell and Bickhard, 1986, p. 95f.]. In fact, his move to intensional logic [Apostel, 1982; Piaget and Garcia, 1987] his concept 'local' necessities [Piaget, 1977/1986], and his increasing emphasis on reflective abstraction, were, among other later developments, all in varying ways inconsistent with his structuralist conception of necessity [Campbell and Bickhard, 1986] - e.g., an intensionally understood 'local necessity' does not have the fully equilibrated closure properties of structural necessity: such an understanding is not a structure at all: 'at the preoperational level ... islets of necessities are already constituted, but they are local and not tied together into stable systems' [Piaget, 1977/1986, p. 236]. Such inconsistencies with his structuralist conception of necessity are, in turn, inconsistent with his structuralist approach to the nature of knowledge in general.

Such later divergences from Piaget's original and basic structuralism can lend themselves to the interpretation that that structuralism does not involve the representation by correspondence that has been claimed here. There seem to be at least three levels of such noncorrespondence' interpretations of Piaget, and I would like to address each briefly. First, it might be claimed that Piaget's structures do not themselves exist except as possibilities of action of some more basic level of mental ontology. Against this: (1) Piaget never says anything like this nor anything about what such a more basic level might be; (2) Piaget presupposes the reality of structures in virtually every discussion of them; (3) Piaget is logically committed to their existence in that, without structures, there is no locus for the process of equilibration; (4) epistemic structures that equilibrate are clearly derived from anatomic structures that physiologically function, and (5) we find such statements as 'If structures are systems that are or seem atemporal, it may be that they do not exist in the child's mind but are merely the product of interpretations made by psychologists ... This is somewhat like stating that even though children are aware of eating and breathing, their stomachs and lungs only exist in the minds of physiologists. ... What the child considers possible, impossible, or necessary furnishes the best proof of the existence of structures in the child's mind' [Inhelder and Piaget, 1980, p. 22f.]. This not only makes the existence of structures very clear, it also exemplifies a tendency in Piaget's writings to assume that to every differentiable function there is an individuated element or structure (or element of a structure) to serve that function - the basic premise seems to be that differentiable (physiological) functions are served by differentiated (anatomical) structures. Further, even if structures *were* themselves only potentialities, the basic point of representation by correspondence remains: if the elements that represent by correspondence need to be constructed each time they are needed (they are only potential), instead of having a persistent existence, that does not alter the basic logic of representation by correspondence.

Second, it might simply be denied that structures are to be taken as representing by correspondence. Against this: (1) Piaget says as much many times, using terms such as 'correspondence' and 'isomorphism'; (2) he presupposes it ubiquitously in, for example, the individuation or atomization of transformations, coordinations, and possibilities, or in the extraction and reinsertion of operations from a structure, and (3) Piaget neither develops a general critique of representation by correspondence, nor does he develop any alternative - he is committed to correspondence by default, as well as by statement and presupposition [see also Kitchener, 1986].

Third, and last, as a stronger version of number two, it might be claimed that, despite appearances, all Piaget needs for the structures, and perhaps all he intended, and certainly all he *needs* to have intended, is 'correspondence' in the metaphorical sense of functional fit with, functional appropriateness to, competence for - as in 'stomachs "correspond" to food'. This claim would maintain Piaget's emphasis on action and transformation, and some of his understanding of the centrality of potentiality. It could also accommodate Piaget's constructivism, and clearly it would not be subject to the objections based on a correspondence model of representation. But this interpretation would render Piaget a naive pragmatist with no model of representation at all. The basic issue here is precisely how representation emerges in or from functionally successful systems. Piaget's model of that emergence is in terms of the correspondences between knowledge structures and the potentialities which they represent. If those correspondences are reinterpreted so as to deny or ignore the emergence of representation and return to mere functionality, then Piaget is deprived of any model of representation, of understanding, at all - of objects, of arithmetic, *or* of necessity. This is simply not a valid interpretation - it loses most of what Piaget was concerned about. Among other losses, it renders Piaget's arguments for endogenous groping completely nugatory and mysterious: if simple functional success is the only consideration, then the crudely approximative utilitarian successes of unguided mere random variations and filtering selections would suffice. Again, in the absence of any developed alternative, Piaget is committed to correspondence not only by what he says and presupposes, but also by default. These considerations make Piaget's later divergences from such a structuralism all the intriguing, but those divergences do not constitute an alternative to representation by correspondence, and, in any case, have no bearing on Piaget's original structural reasonings and motivations in the development of various parts of his theory, such as the epistemic necessity of endogenous groping.

However logical necessity is arrived at, whether by emergence or exquisite complexity, and whatever the significance of Piaget's later partial deviations from a pure structuralism, within Piaget's structural correspondence perspective the necessity for endogenous teleonomic activity is fundamental, and, therefore, so also is the importance of explaining how it works, or at least could work. Furthermore, this teleonomy requires integrated and mutually coherent explanations at both the level of biological evolution - an explanation of phenocopy - and at the level of the individual - an explanation of teleonomic equilibration. Several of Piaget's later works dealt precisely with these problems [Piaget, 1978, 1980, 1985].

Filtering Selections

For the most part, these works attempt to fill in details and respond to criticisms of earlier works such as *Biology and Knowledge* [Piaget, 1971b]. To that extent, they do not affect the general analysis being presented here. There is one set of developments, however, that I will address. First, some continuities: Piaget remains deeply committed to a view of knowledge moving from exogenous formations to endogenous formations [Piaget, 1980, p. 80], via phenocopy in biology, and via 'cognitive phenocopy', or teleonomic equilibration, in development. His major concerns are to show that such progressions happen, and how they happen. Piaget's focus on logical necessity remains [Piaget, 1977/1986, 1987], although the *biological* problem of the emergence of logical necessity recedes somewhat. Within this framework, however, Piaget states

> I would like to make a remark about how [these ideas concerning phenocopy] relate to ideas put forward in an earlier study, *Biology and Knowledge.* The theories expounded in that book seemed, to some colleagues, to be tainted with Lamarckism. Such a suggestion probably arose from the principal assertion which was insisted upon throughout: that there was an inherent improbability in ascribing extremely well-organized and extensive powers of synthesis to the genome itself, unless the many regulating forces involved supply it with 'feedback' information as to the success or failure of endogenous developments. The earlier study was, however, lacking in precision; the present work is intended to provide this. It seems quite evident that the information feedbacks referred to need comprise no 'message', properly so called. ... Feedback need consist only in the progressive and retroactive repercussions (by selective obstruction or blocking) occasioned by a loss of equilibrium. In other words, the supposed message may consist of a *noncodified* indication that 'something isn't working.' When everything is functioning normally, on the other hand, there will be no need for any such indication [Piaget, 1980, p. 9f.].

The significance of these remarks stems from the fact that, in *Biology and Knowledge,* Piaget suggested that the deep reason for the necessity of the truly active and guiding epistemic system is that only with such endogenous constructive activity is the importation of truth, the internal construction of structural correspondences with the world, possible. In particular, only with such an endogenously active system could we expect anything more than mere approximative utilitarian solutions. The above quotation, however, might be taken to suggest that Piaget has moved back from that position. If the only feedback involved is that 'something isn't working', with no message at all beyond that, then it is not dear how the importation of structurally correspondent truth is supposed to occur. It is not dear how this differs from mere 'filtering' or 'sorting' selection, which cannot produce, among other things, logical necessity according to the earlier presentation. If the claims of the importation of truth as an aspect of the endogenously active epistemic system were abjured, then Piaget would be in the severely weakened position of arguing for the active movement from exogenous formation to endogenous formation, especially in the evolutionary form of phenocopy, *solely* on the grounds that it is more efficient than mere random variation and selection. In *Biology and Knowledge,* Piaget suggested that such endogenous activity was *necessary* for the emergence of logical necessity, because logical necessity required the importation of truth, the construction of full correspondences, and that required endogenous constructive activity. If the importation of truth is relinquished, then Piaget's argument from necessity is also, and he is left with a mere efficiency argument. Such an argument would still carry weight, which is not to say that it would be correct, with respect to the specifics of evolution, but it would no longer have such a strong relevance to epistemology, since it then becomes an argument that something is a certain way *merely* because of reasons of efficiency, and no longer for reasons of ontological necessity. Such an argument, if valid, would not be unimportant, but it would have a different import: it would claim a point based on contingent efficiency rather than on epistemological necessity. More deeply, a retreat from structuralist importation of truth would leave utterly unclear how Piaget would explain the epistemology of logical necessity within such a changed framework. Still further, it would render a shambles the relatively tight interconnections of motivation and logic developed in Piaget's writings to this point. The stakes in this comment of Piaget's, then, are not trivial.

This potential import of the comment, however, is contradicted by the insistence in the debate with Chomsky that mere survival adaptation could not possibly produce logical necessity, that adequacy adaptation is required instead. Furthermore, we find

> there are two problems to be distinguished. The first is that of global adaptation or survival (by which I is meant the favorable reproduction of the species or population as much as the survival of individuals). The second is that of the differentiated adaptation which I propose to call adequation. This presupposes a detailed correspondence or morphism (in the mathematical sense) between particular organs or movements of the organism and specific aspects of the environment or of objects affected by the action in question [Piaget, 1978, p. 28].

Here we have not only the critical distinction between survival adaptation and adequation adaptation, but also the explicit claim that adequation adaptation yields structural correspondences from within the organism to the world. That is, we still have the distinction between Piaget's constructivist views on the origin of knowledge, and his structuralist views on the nature of knowledge. Both in *Biology and Knowledge* and in the debate with Chomsky, the claim was made essentially that only with such an active constructivism could full structuralist correspondence be obtained, and only with such full structuralist correspondence could logical necessity emerge.

According to Piaget's [1980, p. 10] disclaimer of any feedback other than simply 'something isn't working', however, such structuralist correspondence truth importation must be possible even with mere functional selection, as long as the epistemic system is endogenously active in its explorations and constructions. In other words, Piaget is accepting a mere filtering or sorting selection, so long as the system constructions that are being selected are themselves heuristically and actively guided. Actively guided variations and mere filtering selections are sufficient for structuralist correspondent knowledge. What Piaget is clarifying here is that (in spite of his denunciations of 'mere sorting' or 'mere filtering') it was always, for him, the active, groping, teleonomic constructions that were important, not the filtering per se: 'importation of truth' does not require 'messages' from the environment, only active groping construction and feedback.

It is not clear, however, that Piaget ever made good on the claim for structuralist correspondent knowledge. The form of his discussions was to explicate necessity in terms of perfect structuralist correspondence, and to argue that endogenous groping was necessary for the construction of such perfect truth importation, without ever explicitly addressing whether structuralist correspondence per se was itself possible or meaningful · There are, in fact, logically deep grounds for concluding that it is impossible to make good on *any* such claim [Campbell and Bickhard, 1986]. It remains a separate and parallel structuralist assumption alongside of Piaget's actively guided constructivist assumption. It remains an unexamined non-sequitur from his arguments concerning the constructivist origins of knowledge.

Two Difficulties: Necessary Epistemic Activity, and Structuralism

At this point, we face two fundamental difficulties: one arising from Piaget's logical commitment to the necessity of an already existing active epistemic system in order for such a system to come into being, and another difficulty concerning his structuralist assumption concerning the nature of epistemic content. The two points are related in that the argument for the first point - the necessity of an active guiding epistemic system - is that such a system is required for the second - structuralist knowledge - to emerge. If, however, it should be the case that structuralist knowledge-by-correspondence is impossible or incoherent on any account, then Piaget's argument for the *necessity* of the endogenously active epistemic agent disappears, and so also does the infinite regress concerning the possibility of the initial emergence of an epistemic system. This way out, however, was not available to Piaget - his structuralist assumptions were too deeply embedded, and it seems highly unlikely that he saw the dilemma that his joint positions forced.

Furthermore, even without Piaget's structuralist assumptions, there are powerful constraints preventing Piaget from giving up the logical necessity of the endogenously active and guiding epistemic system. The first is simply that he had devoted a great deal of effort to the development of a model of phenocopy at the level of evolution, and to abandon the necessity for active 'groping' would have undercut the rationale for those efforts. Logically more fundamental, however, is that Piaget actually was in fact working from a variation and selection model, in spite of his earlier attacks on them. His version, however, had not only the constructive variations, but also the supposedly necessary heuristically guided 'groping' in those constructive variations.

The import of this is that, although Piaget had two logically separable aspects to his constructivism - (1) the constructive variations and 'mere' filtering selections per se, and (2) the endogenously active groping of those constructions - he only had one ontological realm within which to try to understand and model them: the realm of structural equilibration. I have already argued that when Piaget incorporated adaptability into the system of adaptedness, he constrained himself within one ontology of system process, and thereby eliminated the possibility of adequately differentiating between adapted process and adaptational metaprocess - he was logically forced to postulate that 'equilibration' can somehow do both. Here we find a second, similar consequence of this ontological restriction *within* the 'meta'-realm of adaptational metaprocess: with only one level of system process ontology, Piaget could not model the emergence of *separate* guiding constructive metaprocess heuristics within the *general* ontology of (initially random) constructive metaprocesses. He could not model the emergence, the construction, of guiding construction heuristics by processes of initially random constructions and selections. If Piaget had modeled the *initial* systems as systems of random variations and selections, and, therefore, as *producible* by random variations and selections, then the logical impossibility of the emergence of such systems would have been avoided. He could not ground the emergence of active epistemic agents on such initial random processes, however, without giving up the assumption of the logical necessity of active groping construction heuristics, and, more fundamentally, he could not even address the possibility that such active groping might not be initially necessary so long as he had only one system level ontology: one single level of system ontology (active structures) not only forced the merger of adapted process and adaptational metaprocess, it also forced the merger, within the realm of adaptational metaprocesses, of adaptational 'construction' and adaptational 'active groping construction'. Equilibration was forced to do it all.

In addition, Piaget had supporting rationales for contending the necessity of endogenous activity. For example, he held that reason does not change without reason, and equated randomness with irrationality. Random change, then, was not rational change, and could not account for the reasonable change of reason itself; conversely, if reason *did* change without reason, then by virtue of such irrational change, reason would not be reason [Kitchener, 1986, pp. 180f., 191,194]. Note, however, that this rationale is not an argument, but rather a contention appealing to an intuition that reason cannot emerge from something that is not itself reasonable. Whether or not reason can so emerge, however, e.g., from randomness, is precisely what is at stake in this discussion, so Piaget's appeal here is circular. Note further that this rationale yields its own slight variant on the infinite regress of origins: if reason can only be constructed by reason, then we must have reason before we can get reason.

A Way Out

The dilemma is that, within Piaget's perspective, necessity cannot be explained without structuralist knowledge, and structuralist knowledge cannot be obtained without endogenously guiding constructivist epistemic systems, and such epistemic systems cannot come into being without *prior* such systems already existing to engage in the necessary prior endogenous activity. The confrontation of structuralism with the epistemological problem of logical necessity has yielded an aporetic infinite regress of possible origins. There are, however, independent reasons to question structuralist conceptions of knowledge. If some non-structuralist understanding of knowledge were available that could nevertheless explain the emergence of logical necessity, then it might also avoid the perplexity of the emergence of epistemic systems. The difficulty with the postulate of endogenously active epistemic systems does not arise from the assumption that such systems exist, nor even from claims that such activity might be necessary for the origin of *some* sorts of knowledge, but, rather, from the assumption that such endogenous guiding activity is *logically* necessary for the construction of *any* epistemic content - the difficulty is that, with such an assumption, endogenous guidance has to be there before such endogenous guidance can emerge. So, what is required is a perspective on the nature of knowledge and representation that is consonant with the basic constructivist insights of Piaget; that can emerge in a non-guided manner, but is consonant with the potential later development of guided constructivism; and that can explain the emergence of logical necessity without encountering the problems of structuralism.

The claim to be made here is that a partial but essential intuition for such a perspective is already to be found in Piaget's writings: his emphasis on the interactive, operative, character of knowledge and representation. Throughout his life, Piaget emphasized that static figurative representation could not suffice to explain an organism's knowledge of the world - that such figurative representation must be subordinated to the interactive knowledge of how such figurative points could be transformed from one point to another. Without being embedded in such interactive knowledge of possible transformations, representation of a static figurative condition in isolation would constitute essentially no knowledge at all: '...human knowledge is essentially active. To know is to assimilate reality into systems of transformations' [Piaget, 1970, p. 15].

Subordinated or not, however, for Piaget figurative knowledge is an irreducibly separate form of knowledge from operative knowledge - it is intrinsically a representation of structuralist correspondences. Furthermore, even Piaget's concept of interactive knowledge of possible transformations is construed in a structuralist correspondent manner: his concepts of scheme and operation are knowledge structures that correspond to structures of possible transformations [Bickhard and Campbell, 1986; Campbell and Bickhard, 1986; Kitchener, 1986, p. 107; Piaget, 1970, p. 15, 1977, p. 18]. Aside from being a non sequitur from Piaget's constructivist arguments, the concept of knowledge as being foundationally constituted by such correspondences involves a deep logical incoherence. Structuralist representations constitute knowledge by virtue of the known correspondences between those structural representations and structures of potential transformations, but, in order for such correspondences to be known, the potential transformations to which they correspond must already be known: structural representations are essentially copies of structures of transformational possibilities, but, in order to construct a copy of something, you must already know what is to be copied. Structuralist knowledge, then, figurative, transformational, regulative, and operative alike, cannot be an irreducible form of knowledge - it is at best derivative from some other form of knowledge with respect to which such correspondences can be defined.

Piaget was well aware of this impossibility of copy theories of knowledge, and argued against them frequently [Piaget, 1970, p. 15], but he failed to realize that his own structuralist conceptions were subject to the same arguments [Bickhard and Campbell, 1986; Kitchener, 1986, p. 107]. He failed to realize the true scope of such arguments: he directed such arguments against models of the passive, 'copy', *origin* of such knowledge, and in favor of necessarily active constructive origins, without recognizing their validity against the assumption of the structuralist, correspondent, *nature* of knowledge. Of particular current relevance, the infinite regress of needing endogenous epistemic groping to explain the origin of endogenous epistemic groping is an evolutionary variant of Piaget's own psychological argument against copy theories: we must already have knowledge of something in order to produce a copy of it, and, if knowledge is presumed to be *constituted* as copies, then we must already have a copy in order to construct a copy, we must already have knowledge in order to explain the origin of knowledge.

Piaget's argument against copy theories of representation is in fact a version of the classic skeptic's argument: how can we know whether our representations are correct, since, in order to check them, we would have to have epistemic access to what they represent that is independent of those very representations, and that, by definition, is impossible. A variant of that argument points out that it is impossible even to specify what the correspondences are *supposed* to be with except in terms of some other representation, and that such dependence on some other representation for their very definition makes such representations by correspondence necessarily derivative - they cannot be foundational, else there would be no other representation by which their representational content, their epistemic correspondence relationship, could be specified. Representations that represent by virtue of known epistemic correspondences are generically called encodings, and the general family of arguments to which Piaget's argument belongs applies to all forms of encodings considered as logically independent representations, including, in particular, Piaget's own structuralist conception of representation and knowledge [Bickhard, 1980a; f Bickhard and Richie, 1983; Campbell and I Bickhard, 1986].

Furthermore, it applies also to Chomsky and Fodor's in-principle argument for a necessary innatism of representations: we have to already know what we are going to encode in order to define or construct an encoding of it - an incoherence; therefore, we cannot ever get new basic encodings, but only new combinations of already present - therefore ultimately innate - encodings. But this in-principle argument does not stop with development - it yields an impossible infinite regress of origins at the level of evolution just as much as at the level of learning or development. Piaget rightly presses this regress against Chomsky and Fodor's position [Piattelli-Palmarini, 1980], but Piaget implicitly shares the same encoding presuppositions concerning the nature of representation, and is thus subject to variants of the same regress [Campbell and Bickhard, 1987].

If structuralist conceptions of representation are incoherent, if structuralist truth importation or correspondence construction is impossible and incoherent anyway, then Piaget's motivation for the logical necessity of endogenously active 'groping' is broken: such groping was considered necessary for structuralist truth importation, and truth importation was necessary for the formation of logico-mathematical structures, but, if structuralist truth importation is impossible in any case, then the middle link in the argument no longer exists. This frees us to consider the possibility that endogenously active variations evolved and develop from random variations - as long as the constraint to one single ontological realm of systems undergoing equilibration is given up - but imposes the problem of accounting for necessity in some other way.

Peeling away Piaget's structuralist encodingism leaves a purified interactivism - knowledge is the capability of successful interaction. The claim to be made now is that interactivism can account for what Piaget thought required structuralism - the necessity of logic and mathematics - and can do so without giving rise to a logical impossibility of origin. That is, interactivism is consonant with constructivism; does not require structuralism; accounts for logical necessity anyway; does not logically require active guiding variations - and thus avoids the infinite regress of origins; and is, nevertheless, able to model the emergence of such guiding heuristics for variations. The point now is to outline how to make good on that claim.

Interactivism

Interactivism is a perspective on the nature of knowledge and representation. Knowledge is the capability for successful interaction. Representation is the aspect of knowledge which involves differentiating the system's activities in accordance with relevant differentiations of what the system is interacting with - that is, representation emerges from the sense in which a system must be sensitive to what is being interacted with in order for the interaction to be successful: Representation emerges in that an interactive subsystem's internal end state after completion of an interaction serves to *implicitly define* the set of those potential environments that would, if encountered by that interactive subsystem, yield that particular internal final state, and that final state, in turn, can serve to help *differentiate,* select among, the further potential interactions of the overall system. In distinction from Piaget, interactivism views representation as an emergent *functional aspect* of any successful interactive system, not as a matter of correspondences, structural, elemental, or otherwise. It is clear that interactive knowledge is a form of knowledge, but many questions can be asked concerning its nature, its implications, and its adequacy as a full model of knowledge and representation. Most of these questions will not be addressed here [see Bickhard, 1980a; Bickhard and Richie, 1983; 1983; Campbell and Bickhard, 1986 for more extensive discussions]; what will be of concern will be the sense in which interactivism can satisfy the constraints and resolve the perplexities that have been analyzed in Piaget's position.

Convergences with interactivism are deeply embedded in Piaget's intuitions. His arguments against static copy models, for the intrinsically active character of knowledge, for the fundamental locus of knowledge in potentiality, and for the subordination of figurative knowledge to operative knowledge, all manifested aspects of his underlying convergent intuitions. Interactivism, in effect, results from Piaget's position when it is rendered internally consistent with Piaget's own arguments against copy theories of knowledge, and elaborated with an interactivist conception of representation in place of the structuralist conception [Bickhard and Campbell, 1986]. Interactivism, then, is consonant with Piaget's core intuitions about knowledge.

Interactivism is also consonant with Piaget's constructivist intuitions concerning the origins of knowledge: interactive systems can be constructed just as much as representational structures can. There is more than just a consonance here, however: interactivism logically forces constructivism. Piaget continually argued against the model of knowledge as being passively impressed on the system by the environment, and yet a major appeal of such a conception derives directly from the encodingism of Piaget's own structuralism. If internal representations are such by virtue of their correspondences with the world, then it would seem to make sense that they might arise by being impressed on the system by the structures in the world - with the classic tabula rasa as the paradigmatic case. In spite of this consonance with his own structuralism, Piaget understood that such passive copying from the world was not a satisfactory model. This understanding of the necessarily active nature of knowledge acquisition, however, was independent of his structuralism. Interactivism, in contrast, does not represent by virtue of structural correspondence, nor any sort of encoding correspondence, and, therefore, there is no way in which impression from the world could create representation. Interactive representations are such by virtue of successful functional implicit definitions and differentiations, not by virtue of correspondences at all. Interactive systems must be tentatively constructed and tentatively improved, modified, and so on, and then retained or changed depending on whether or not the changes contribute to their interactive potentialities: interactivism as a conception of the nature of knowledge forces constructivism by variation and selection as a model of knowledge acquisition. In that respect, it has a deeper coherence than does Piaget's hybrid of constructive knowledge acquisition with structural knowledge character.

Interactivism, on the other hand, does *not* logically require active heuristic guidance in its constructive variations. Random variations and filtering selections are logically sufficient, both to develop an already existing interactive system, and, of particular importance in this context, to yield the emergence of primitive interactive systems de novo. Interactivism, thus, avoids the aporia of possible origins that Piaget's position creates. In particular, whatever the facts of the matter may be, interactivism does not logically require anything like phenocopy at the biological level.

On still another hand, however, interactivism can account for the development of subsidiary heuristics for the guidance of its problem definitions and variational constructivist tries at solutions. Since Piaget explicitly rejected this possibility - 'Now, it is this teleonomy that has to be explained because ... it could [not] result from a mere sorting' [Piattelli-Palmarini, 1980, p. 281] – I will devote a little more attention to outlining how this could occur. What Piaget seems to have missed is that a random variation selection meta-process is, in a logical sense even though not necessarily in a physical sense, a part of the *environment* for the interacting system, so that the variation and selection meta-process will have a tendency to improve the system's interactions with those same *variations and selections* that is logically very similar to its tendency to improve the system's interactions with the environment. In other words, the system will tend to learn to make use of the variation and selection meta-process. If the variations are simple behavioral trials, as Piaget probably had in mind when he initially developed his opposition to trial and error models, then there is no way in which this point can hold. But if the variations are themselves constructions of goal-directed problem-solving systems, then those systems will improve in their *general* goal-reaching and problem-solving heuristic capabilities. One important potential heuristic, or type of heuristic, in turn, will be for a problem-solving heuristic to initiate an interaction that the system cannot currently successfully complete, *so as to explicitly invoke the constructive variation and selection meta-process* - a heuristic that makes use of the possibility of random variation and selection as one of its resources. In this manner, one important ontological realm of processes that will tend to emerge - and to become itself an important realm of farther development - is the realm of those processes that heuristically differentiate problem types, and heuristically try out solution types appropriate to those problem types, always within the outer context of less and less heuristically guided, less and less 'knowledgeable' strategies that might be used if others do not work, with pure random variations in system organization as the outermost 'margin' of available 'heuristics'. That is, one important potential type of heuristic will be to explicitly and endogenously invoke groping interactions with the environment.

A problem-solving interactive system with a random variation and environmental selection meta-process, then, will tend, as an overall system-plus-meta-system, to develop precisely the gropings that Piaget felt had to be independently assumed from the start. At this level of analysis, such gropings, and especially the development of such gropings, will be very inefficient, but with the further evolution of emotions and consciousness, the efficiency and effectiveness (and ubiquitousness) of the development of such groping heuristics will increase enormously [Bickhard, 1980b; Campbell and Bickhard, 1986]. Interactivism, then, does not presume endogenous gropings from the beginning, but it can account for their potential evolution and development later on.

What about Necessity?

I have shown how interactivism captures one of Piaget's deepest intuitions; avoids the incoherence of structuralism; has a deep consonance with constructivism - a logical forcing of constructivism, in fact; does not assume primordial endogenous gropings - and thus unconfounds adaptedness and adaptability; and yet can account for their later emergence. I have shown, in other words, how interactivism solves and dissolves the problems and perplexities that have been uncovered in Piaget's position, except for the problem of the emergence of logical necessity. This topic is developed extensively elsewhere [Campbell and Bickhard, 1986], and will not be repeated here, but I do want to outline it sufficiently to point out some basic principles that are involved.

First, some preliminaries are required. The interactive perspective gives rise to a hierarchy of levels of potential knowing that are essential to the following discussion. Simply, a first level interactive system that interacts with and in that sense knows the environment will have properties as a system that cannot themselves be (interactively) known from within that system, but which could be known from a second level of system interacting with and knowing the first level in the same sense in which the first level knows the environment. That second level, in turn, will have properties that can be known from a third level system, and so on.

With a constructivist model of development, such a hierarchy of potential levels of what can be known forces a corresponding sequence of stages of development - no system at a given level of knowing can be constructed unless there are already systems at the next lower level in place to be known. The hierarchy, then, must be developmentally ascended, if at all, one stage at a time in hierarchical sequence. Such knowing level stages have strong affinities with Piaget's stages, especially his post-1970 conception, but also some distinctive differences. For example, knowing level stages do not involve any particular structural or organizational principles as definitive of the stages - there are no structures of the whole. Correspondingly, development through the stages may be widely out of phase across varying domains of development. Furthermore, the knowing level stages are half a cycle advanced compared to the Piagetian stages with level one extending from birth to an proximately four, level two to around nine and so on; the attainment of knowledge of the major invariances at each level - e.g. object permanence at level one, the conservations at level two, etc. - is a major accomplishment *within* the levels of knowing stages, instead of demarcating the boundaries between stages as for Piaget. Each higher level relates to, and develops out of, the next lower level via reflective abstraction - similarly to Piaget's 1970s model [Piaget, 1977/1986, 1985]. This simple list of properties of knowing level stages is for illustrative purposes - the model is extensively elaborated in Campbell and Bickhard [1986] - but the existence of the levels, the stages, and the reflective abstraction relationship between adjacent stages are needed for the following discussion outlining the interactive model of the development of logical necessity.

The first distinction that needs to be made in presenting this model is that between the development of representations of relationships that *are* logically necessary, and the development of representational knowledge *of* that necessity. Piaget persistently conflated this distinction: he persistently conflated models of the representation of relationships that are in fact logically necessary with models of the representation of that property of logical necessity. This conflation was itself forced by Piaget's structuralism: structures represent by virtue of morphisms with what they represent; they represent abstract relationships by embodying those relationships; they represent, in particular, logical necessity by *being* (closed mathematical structures with) logically necessary (properties). Thus there is no possible model of the distinction between *being* logically necessary and *knowing* that necessity. In the knowing levels model, representations of relationships that may *be* in fact logically necessary will occur at some appropriate level of knowing, but that *property* of necessity will be a property implicit in the representations at that level, and, as such, *will only be knowable from the next higher level - in general via reflective abstraction*. There are, therefore, two parts to the knowing levels model of the development of logical necessity: how could logically necessary relationships come to be represented (rather than just approximate relationships, merely sufficient-to-survival), and how can that property of logical necessity come to be represented? How the representation of logical necessity develops is complex, interesting, and important, but it is not the problem that is central to the current discussion [see Campbell and Bickhard, 1986, for the development of the representation of logical necessity]. Piaget conflated relationships that are in fact logically necessary with knowledge of that necessity, but his strongest arguments proceeding from the epistemological problem of logical necessity concerned the development of relationships that in fact *have* logical necessity, independent of whether or not that necessity is represented or understood - Piaget was concerned with the development of logico-mathematical structures that are intrinsically logically necessary (closure), instead of simply being approximate and sufficient to survival. He was concerned with how randomness could possibly yield logical necessity at all. That is the issue that will be addressed at this time.

The Representation of Necessary Relationships

The general form of the argument for how this could occur is, with some irony, illustrated most strongly within psychology by Piaget's own model of stage sequences. In particular, if formal operations are in fact operations on concrete operations, then the explanation of why they would occur in the sequence in which Piaget proposed is not a contingent explanation at all. It is an explanation by virtue of logical necessity: formal operations follow concrete operations (if they exist at all) because that is a logically necessary consequence of what they are; that sequence is a logically intrinsic property of the ontology of those stages. No process, random or not, could construct those stages and violate that sequence, because that sequence is intrinsic to what the stages are - formal operations-on-operations *cannot* precede concrete operations per se. Such arguments by intrinsic necessity are unusual in psychology (though common in, for example, physics), and are not generally recognized as a distinct legitimate form of argument and explanation [see Campbell and Bickhard, 1986, on intrinsic constraints]. Piaget did recognize this form of argument, and provided the prime example in the dominantly empiricist field of psychology, and yet did not seem to recognize its potential relevance to one of his very special problems - the development of logical necessity. Just as the sequence between concrete and formal operations is intrinsic to what they are, and thus is logically necessary (this point holds independent of whether or not such stages actually model human development), so also may other relationships between representations be intrinsic to those representations, and thus logically necessary. As already mentioned, such properties of intrinsicness, of logical necessity, will only be *knowable* from the next level of knowing, but the necessity will already be intrinsic to, and implicit in, the representations at the lower level.

From the interactive perspective, then, the mystery dissolves. Representations may be developed at a given level of knowing (perhaps even randomly developed) that in fact are related in intrinsically necessary ways, and those properties of necessity may come to be represented and known at the next higher level. The development of necessity appeared deeply problematic to Piaget in part because of the conflation between necessity per se and knowledge of necessity, but even more so because he assimilated necessity to his arguments from design - how could something so perfect as logical necessity arise from mere randomness? Random variations and selection, however, explore a space of constraints, however randomly (or derivatively guidedly), *that include both contingent constraints and intrinsic constraints,* and will develop interactive representations of the one just as much as of the other. Explorations at higher knowing levels may then discover the distinction between the contingent and the necessary, and the special invariance properties of necessity - invariance under organizations of counter-factuals.

Tendencies toward Necessity

There is, however, one additional part of the puzzle for Piaget. It is not just that epistemic systems do (contingently) in fact develop logically necessary organizations and knowledge of that necessity; it is, in addition, that such developments seem to be a deep tendency of life and of human beings. The additional part of Piaget's question is, not just how could such developments occur at all, but how and why is there such a tendency for their development?

There are two senses in which such a tendency toward necessity could exist: as a directed heuristic tendency, and as an intrinsic emergent tendency. We will find both forms and will address the heuristic tendency first. The first step in understanding the heuristic tendency toward necessity is to recognize that logical necessity is a form of invariance, and invariance is intrinsically useful, and therefore, will tend to be developed - e.g. objects and conservations [Bickhard, 1980a]. An intuition of the intrinsic usefulness of invariant properties and relationships can be derived from consideration of the point that invariance means scope and extension of application of those properties and relationships. Invariance is unchangingness, stability, with respect to other changes of certain sorts, and, therefore, scope and extension of application across the spaces of those sorts of changes. Logical necessity, being an invariance across certain sorts of unbounded counterfactual transformations, in Piaget's words, a permanent or perfect equilibrium, an unchangingness with respect to a space of *logically possible* transformations. Another perspective on this same point is to realize that invariance with respect to some class of transformations is equivalent to being abstracted across that space of transformations, being in common across, being a form common to, such a space of transformations.

Being an abstract form common to a space of counterfactual logical transformations is precisely the way in which (though not precisely the terms in which) Piaget characterized logico-mathematical structures when he drew the analogy between genomes, and such structures: logico-mathematical structures are precisely the general forms that are common to, that underlie, all knowledge. 'Any kind of knowledge about an object is always an assimilation into schemes, and these schemes contain an organization, however elementary, which may be logical or mathematical' [Piaget, 1971b, p. 335] and 'physical and experimental knowledge cannot possibly be established without some structuration and logico-mathematical framework' [Piaget, 1971b, p. 342]. Logical necessity, then, is invariance in one of its deepest senses, and, therefore, implies potential scope of framework in one of its most general forms. The *heuristic* tendency for the development of logically necessary relationships, in turn, derives naturally from the tendency to develop functional interactive systems, and to develop heuristics and frameworks for the development of such systems. (This is prior to the influence of esthetic appreciations in the explicit history of mathematics - such appreciation in effect explicitly seeks ever deeper relationships and organizations. The interactive model of esthetic appreciation, and related phenomena such as the search for 'deeper connections' in science [Shapere, 1984], requires separate extensive presentation.) In sum, logically necessary organizations are an ultimate form of invariance, and invariances have a tendency to be functionally useful, and, therefore, life, and human beings in particular, have a tendency (to develop a heuristic tendency) to develop them.

Interactivism also provides an explication of an *intrinsic* epistemic tendency to develop necessity, in addition to such a heuristic tendency. This tendency, also derived from the explication of logical necessity as an ultimate form of invariance, converges with Piaget's intuition derived from necessity as (locally) perfect equilibrium [Piaget, 1977/1986; Kitchener, 1986, pp. 56-58]: If necessity is constituted as perfect equilibrium, then equilibration, in tending toward 'better' equilibrium, will naturally tend toward 'perfect' equilibrium, and will hold there, be stable, when perfect equilibrium - necessity - is attained. Similarly, if variations and selections are invoked by dysfunctionalities in interaction with the environment, then variations and selections will tend toward system organizations that are functionally stable, invariant, with respect to spaces of such invariances are attained. Logical necessity, as an ultimate form of such invariance, will participate in this tendency.

Note that the intuition of necessity involving an epistemic exhaustion of a space of possibilities is common to Piaget's model and that of interactivism, but, for Piaget: (1) that exhaustion is of the form of a point- by-point structurally representational correspondence to those possibilities; (2) the possibilities themselves are only of transformations or coordinations, and (3) necessity emerges - somewhat mysteriously - in the 'closure' of such a structure. For interactivism, on the other hand: (1) that epistemic 'exhaustion' is an invariance, an exceptionlessness, across the space of possibilities, which invariance can *hold* or be true *with or without* being represented or known by the system - and certainly without any correspondences to those possibilities, structural or otherwise (what becomes of the structuralist model if the domain of relevant possibilities is infinite?); (2) the possibilities involved are limited only by the *implicit* representational powers (stage) of the system in the case of unrepresented necessities, or the *explicit* representational powers in the case of represented necessities - they are not limited to transformations, coordinations, or mathematical structures, but could include, for example, spaces of possible facts, possible worlds, possible arguments, possible situations, possible ontologies, and so on, and (3) necessity *is* precisely such exceptionlessness or invariance - intuitively convergent with logical necessity as 'true in all possible worlds', but without any commitment to the ontological baggage of that 'possible worlds' approach to modality - and has no intrinsic relationship to anything like Piaget's transformational or mathematical 'closure'. Among other consequences, interactivism can easily accommodate Piaget's 'local necessities' - many, if not most, necessities are 'local' invariances, and not just at preoperations - whereas Piaget's structuralism cannot. Furthermore, the interactivist approach generalizes easily to consideration of different *kinds* of necessity corresponding to differing *domains* of invariance, e.g., pragmatic necessities, physical necessities, logical necessities, ontological necessities, moral necessities, or existential necessities.

The tendency toward necessity is an *intrinsic* tendency for an interactive epistemic variation and selection constructive system. It is an intrinsic product, an emergent, of the relationship between epistemic variations and selections and the nature of necessity as an invariance. As such, it is prior to the development of values [Campbell and Bickhard, 1986] for necessity, and to the development of heuristics for the attainment of logical necessity - and, as such, it can ground the development of both. That is, the intrinsic tendency toward necessity can ground the development of the heuristic tendency [still further, there will be an intrinsic tendency for the *development* of the values and heuristics of the heuristic tendency, not just a 'grounding', a making possible, of that heuristic development - see Bickhard, in prep.].

This distinction between an intrinsic tendency toward necessity, and a heuristic tendency toward necessity, is one that Piaget cannot easily make: the structures are necessarily groping, and the tendencies toward necessity of the groping are necessarily both intrinsic and heuristic. Again we see the constraints of having only one ontological level of active structures.

Interactivism, then, differentiates and explicates both the heuristic and the intrinsic tendencies toward necessity in epistemic systems. It solves both the problem of how contingent and random process could yield knowledge of necessary relationships, and the problem of why there is such a deep tendency to do so.

Summary and Conclusion

This analysis began by considering some of Piaget's earlier objections to trial and error models of development and evolution, and then tracing forward some of the ensuing developments in Piaget's thought. Along the way, a coherence was found among Piaget's structuralism, his commitment to the necessity of endogenously groping guidance, his commitment to phenocopy, his commitment to a single ontological realm of equilibrating structures, his conflation of adaptedness and adaptability, and his conflation of representation of logically necessary relationships and knowledge of such necessity. This set of perspectives and commitments is strongly interconnected by motivational and logical ties: Structuralism, in addressing necessity, yields a model of logico-mathematical knowledge as perfect correspondences; perfect correspondences, in order to be attained by a constructivism, yield the necessity of active groping; epistemically necessary active groping yields phenocopy at the level of evolution, and the infinite regress of origins at the level of logic - these are the fundamental errors. In addition, structuralism yields a single ontological realm of equilibrating system, which forces a conflation of adaptedness and adaptability, and a conflation of the representation of logically necessary relationships with the representation of their necessity - these are supporting errors that made it difficult, if not impossible, to see past, or to recognize at all, the fundamental errors. Each one of these positions is in error, but, because of their tight interconnections, they can be peeled away from the rest of Piaget's model fairly neatly.

What remains, I have argued, is a more deeply coherent intuition of interactive knowledge, which, when supplemented with *interactivism,* yields necessarily constructivist origins of knowledge, and plausible accounts of the development of endogenous developmental heuristics and of logical necessity. The core of *interactivism* is a model of the emergence of a noncorrespondence form of representation on the basis of such an intuition of interactive knowledge. Interactivism also yields an intrinsically necessary sequence of developmental stages, and other consequences not directly on the path of this discussion.

The claim, then, is that interactivism captures and coherently derives the deepest of Piaget's insights, and correctly differentiates between his valid and invalid commitments, h particular, interactivism joins Piaget in addressing the epistemological problem of necessity, and offers a solution to that problem that avoids the deficiencies of Piaget's structuralism.

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